CS3133

HW#3

DUE: Tuesday, September 13

1. (8 points) For any NFA $N = (Q, \Sigma, \Delta, S, F)$, consider $L_\alpha(N)$ to be the set of all strings z such that there is a path from **every** state of S to **every** state of F. Prove that $L_\alpha(N)$ must be regular.

2. (9 points) Give regular expressions for the following languages over {0, 1}.
   a Strings that begin with 01 and end with 10 and contain 11. Note that 0110 belongs to the language.
   b Strings that contain 01 **and** contain 10.
   c Strings in which every 0 is **either** immediately preceded by a 1 **or** immediately followed by a 1.

3. (8 points) For each of the following assertions about regular expressions, **either** prove that it is correct using the identities and rules (9.1) → (9.18) on pp. 49 → 50 of our text **or** show that it is incorrect.`
   a $1^*(011^*)^* + 1^*(011^*)^* 0 = (1+01)^*(\varepsilon + 0)$
   b $1^*(01^*)^* + 1^*(011^*)^* 0 = (1+01)^*(\varepsilon + 0)$

4. (14 points) For each of the following languages over {a, b, c}, tell whether or not it is regular and justify your response.
   a $\{a^i b^j c^k \mid 0 \le i \le j \wedge 0 < k\}$
   b $\{a^i b^j c^k \mid 0 \le i, j \wedge 0 < k\}$

Solutions to HW#3

1. Given any NFA $N=(Q,\Sigma,\Delta,S,F)$, construct DFA $M=\left(2^Q,\Sigma,\delta,S,\left\{A\in 2^Q \mid A\subseteq F\right\}\right)$ where for any $A\in 2^Q, a\in A$, $\delta(A,a)=\bigcup_{q\in A}\Delta(q,a)$.

CLAIM: $L(N)=L(M)$. **PROOF**: (Want to show that $(\forall z\in\Sigma^*)\hat{\Delta}(S,z)=\hat{\delta}(S,z)$)
$\varepsilon\in L(N)\Leftrightarrow S\subseteq F\Leftrightarrow \varepsilon\in L(M)$. Assume that $\hat{\Delta}(S,x)=\hat{\delta}(S,x)$. Then
$\hat{\Delta}(S,xa)=\Delta\left(\hat{\Delta}(S,x),a\right)=\Delta\left(\hat{\delta}(S,x),a\right)=\delta\left(\hat{\delta}(S,x),a\right)=\hat{\delta}(S,xa)$. So
$\hat{\Delta}(S,x)\subseteq F\Leftrightarrow\hat{\delta}(S,x)\in\left\{A\in 2^Q \mid A\subseteq F\right\}$ and $L(N)=L(M)$.

2. ***a*** $01(0+1)^*11(0+1)^*10+011(0+1)^*10+01(0+1)^*110+0110$

b We note that an 01 precedes a 10 (as in the first two terms) or a 10 precedes an 01 (as in the second two terms).

$(0+1)^*01(0+1)^*10(0+1)^*+(0+1)^*010(0+1)^*+(0+1)^*10(0+1)^*01(0+1)^*+(0+1)^*101(0+1)^*$

c If every 0 is immediately preceded by a 1, then the language is $\left(1^*101^*\right)^*$. If every 0 is immediately followed by a 1, then the language is $\left(1^*011^*\right)^*$. Or two 0's can share a 1, thus 010. So our language is

$$\left(1^*(10+010+01)1^*\right)^*1^* \quad .$$

3. ***a***

$$\begin{aligned}1^*\left(011^*\right)^*+1^*\left(011^*\right)^*0&=1^*\left(011^*\right)^*(\varepsilon+0) &&(9.7)\\ &=(1+01)^*(\varepsilon+0) &&(9.16)\end{aligned}$$

b $1^*\left(01^*\right)^*+1^*\left(011^*\right)^*0\neq(1+01)^*(\varepsilon+0)$ since 00 belongs to the language $1^*\left(01^*\right)^*+1^*\left(011^*\right)^*0$ but not to the language $(1+01)^*(\varepsilon+0)$.

4. ***a*** $\left\{a^ib^jc^k \mid 0\le i\le j\wedge 0<k\right\}$ is not regular. If it were, then there would be an n such that every string $z\in\left\{a^ib^jc^k \mid 0\le i\le j\wedge 0<k\right\}$, $|z|\ge n$, could be written $z=uvw$ with $|uv|\le n$, $|v|>0$, such that $\left\{uv^lw \mid l\ge 0\right\}\subseteq\left\{a^ib^jc^k \mid 0\le i\le j\wedge 0<k\right\}$. Choose $z=a^nb^nc\in\left\{a^ib^jc^k \mid 0\le i\le j\wedge 0<k\right\}$. Any attempt to express z as uvw with with $|uv|\le n$ and $|v|>0$ would have u spanning a nonempty string of as and no other letter. So

$uv^2w \notin \{a^ib^jc^k \mid 0 \le i \le j \wedge 0 < k\}$, which contradicts the conditions of the Pumping Lemma. So $\{a^ib^jc^k \mid 0 \le i \le j \wedge 0 < k\}$ can not be a regular language.

b $\{a^ib^jc^k \mid 0 \le i, j \wedge\ 0 < k\}$ is regular. It is accepted by the DFA $(\{q_a, q_b, q_c, q_{BlackHole}\}, \{a, b, c\}, \delta, q_a, \{q_c\})$, where q_a "means" that the only strings read are a^*, q_b "means" that the only strings read are a^*bb^*, q_c "means" that the only strings read are $a^*b^*cc^*$, and q_{BH} is a *BlackHole* from which one can never escape.

δ	a	b	c
q_a	q_a	q_b	q_c
q_b	q_{BH}	q_b	q_c
q_c	q_{BH}	q_{BH}	q_c
q_{BH}	q_{BH}	q_{BH}	q_{BH}