CS3133

HW#3

DUE: Friday, September 14

1.(6 points) Give a regular expression for the set of all strings over $\{a,b\}$ which contain at least two *a*'s and at least one *b*.

2. (12 points) For any language *L*, we define a new language *Prefix*(*L*) to be $\{y \mid \exists z\ yz \in L\}$. So $Prefix(\{011, 1\}) = \{\varepsilon, 0, 01, 011, 1\}$. Describe an algorithm to accept as input a regular expression α and to construct a regular expression for $Prefix(L(\alpha))$. So upon receiving input $011+1$ your algorithm would return $\varepsilon+0+01+011+1$, or an equivalent regular expression.

3. (6 points) Prove or give a counterexample to the following.

CONJECTURE: Any subset of a regular set is regular.

4. (20 points) Are the following languages regular? Defend your answers.

a $\left\{0^i 1^i w \mid i \geq 0 \wedge w \in \{0,1\}^*\right\}$

b $\left\{a^l b^m c^n \mid l > m > n \geq 0\right\}$.

c $\left\{0^m 1^n \mid m \neq n\right\}$ Hint: Express it in terms of other languages we have seen.

d *LL* for all regular languages *L*.

5. (3 points) Give a regular expression for the language of assignment statements such as $E \leftarrow 2.71828$ or $PI \leftarrow 3.14159265358979323$ An assignment statement is a nonempty string of uppercase letters, followed by a left arrow, "$\leftarrow$", followed by a nonempty string of digits without a leading 0, followed by a decimal point, ".", followed by a nonempty string of digits without a trailing 0.

6. (8 points) Do **PROBLEM 14** on page 320 of Kozen.

7. (4 points) Let $\alpha = (00+11)^*(01+10)(00+11)^*$.

a Give an NFA *N* such that $L(N) = L(\alpha)$.

b Give a $w \in \{0,1\}^*, |w| = 8$ such that $w \notin L(\alpha)$.

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Solutions to HW#3

1. $(a+b)^* aab(a+b)^* + (a+b)^* baa(a+b)^* + (a+b)^* a(a+b)^* b(a+b)^* a(a+b)^*$

2. One solution is to apply the function *Prefix* recursively to regular expression α.

α	$Prefix(\alpha)$
$\emptyset$	$\emptyset$
$a \in \Sigma$	$a + \varepsilon$
$\alpha + \beta$	$Prefix(\alpha) + Prefix(\beta)$
$\alpha\beta$	$Prefix(\alpha) + \alpha Prefix(\beta)$
α^*	$\alpha^* Prefix(\alpha)$

A different solution is to use THEOREM 8.1 of the text to convert α to a DFA *M* such that $L(\alpha) = L(M)$, then convert *M* to another DFA *M'* which is equivalent to *M* except that every state of *M* from which a final state of *M* is reachable becomes a final state of *M'*. $Prefix(L(\alpha)) = Prefix(L(M)) = L(M')$. Finally use THEOREM 8.1 to construct a regular expression β such that $L(\beta) = L(M') = Prefix(L(\alpha))$.

3. The CONJECTURE is false. $\{0,1\}^*$ is regular, because it is accepted by $(\{q\},\{0,1\},\delta,q,\{q\})$ with $\delta(q,0) = \delta(q,1) = q$. But we showed that $\{0^n1^n \mid n \geq 0\}$ isn't regular, even though $\{0^n1^n \mid n \geq 0\} \subseteq \{0,1\}^*$.

4. ***a*** $\left\{0^i1^i w \mid i \geq 0 \wedge w \in \{0,1\}^*\right\} = \{0,1\}^*$, so it is regular. It is accepted by the DFA $(\{q\},\{0,1\},\delta,q,\{q\})$ with $\delta(q,0) = \delta(q,1) = q$.

b $\{a^l b^m c^n \mid l > m > n \geq 0\}$ is not regular. If it were, the Pumping Lemma assures us of the existence of a *k* such that $a^{k+2}b^{k+1}c^k$ can be expressed as *xyz* such that $\{xy^iz \mid i \geq 0\} \subseteq \{a^l b^m c^n \mid l > m > n \geq 0\}$. There are 5 cases to consider:

<u>case: *y* spans only *a*'s</u>. xy^0z does not have more *a*'s than *b*'s, so this is a contradiction.
<u>case: *y* spans some *a*'s and some *b*'s</u>. xy^2z has *a*'s followed by *b*'s followed by *a*'s followed by *b*'s, so this is a contradiction.
<u>case: *y* spans only *b*'s</u>. xy^0z does not have more *b*'s than *c*'s, so this is a contradiction.
<u>case: *y* spans some *b*'s and some *c*'s</u>. xy^2z has *b*'s followed by *c*'s followed by *b*'s followed by *c*'s, so this is a contradiction.
<u>case: *y* spans only *c*'s</u>. xy^2z does not have more *b*'s than *c*'s, so this is a contradiction.

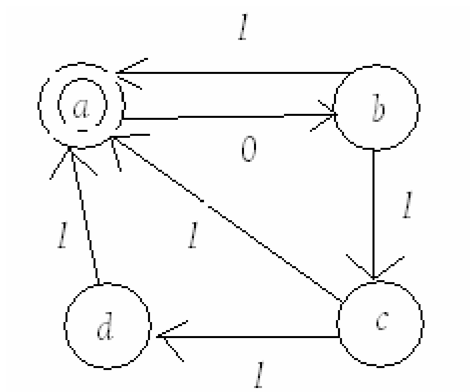
Since each of the five possible cases yields a contradiction, $\{a^l b^m c^n \mid l > m > n \geq 0\}$ is not regular.

c $\{0^m 1^n \mid m \neq n\}$ is not regular. PROOF: We know that the language $L(0^*1^*)$ is regular and that regular languages are closed under set difference. If $\{0^m 1^n \mid m \neq n\}$ were regular, then the fact that $\{0^n 1^n \mid n \geq 0\} = L(0^*1^*) - \{0^m 1^n \mid m \neq n\}$ would contradict our previous result that $\{0^n 1^n \mid n \geq 0\}$ is not regular.

d As we saw in class and on the top of page 22 of our text, the regular languages are closed under concatenation.

5. $(A+\ldots+Z)(A+\ldots+Z)^* \leftarrow (1\ldots9)(0\ldots9)^*.(0\ldots9)^*(1\ldots9)$

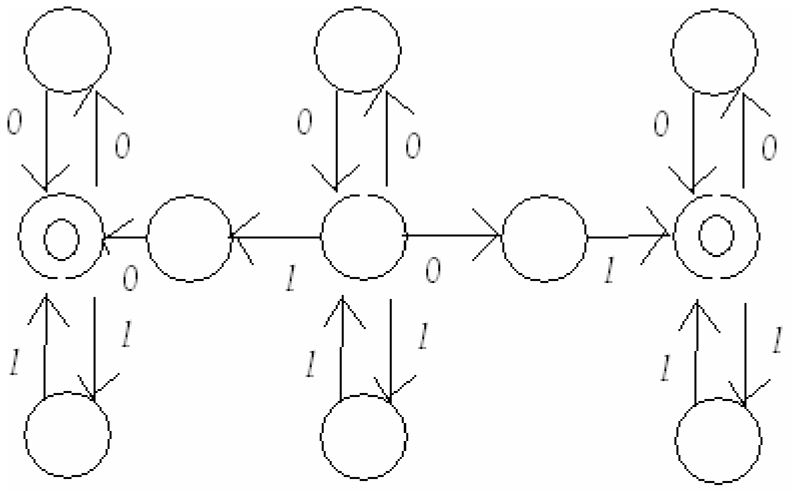
6.



DFA $(\{\{a\},\{b\},\{a,c\},\{a,d\},\varnothing\},\{0,1\},\delta,\{a\},\{\{a\},\{a,c\},\{a,d\}\})$

δ	0	1
$\{a\}$	$\{b\}$	$\varnothing$
$\{b\}$	$\varnothing$	$\{a,c\}$
$\{a,c\}$	$\{b\}$	$\{a,d\}$
$\{a,d\}$	$\{b\}$	$\{a\}$
$\varnothing$	$\varnothing$	$\varnothing$

7. ***a***



b $w = 00000000$