CS3133

HW#3

DUE: Monday, September 12

1. (5 points) Do PROBLEM 1 of HOMEWORK 2 on pg. 302 of our text.

2. (3 points) Give regular expressions describing the following languages over $\{0,1\}^*$:

   a The set of all strings of lengths divisible by 3.
   b The set of all strings containing 000.
   c The set of all strings with an even number of 0's and every 0 is followed by at least one 1.

3. (6 points) ***a*** Give examples of languages L_0 and L_1 such that L_0 and $L_0 \cup L_1$ are regular but L_1 is not regular.

b Give examples of languages L_0 and L_1 such that L_0 is regular but L_1 and $L_0 \cup L_1$ are not regular.

4. (8 points) Prove that $\{a^i b^j c^{2j} \mid i \geq 0, j \geq 0\}$ is not regular.

5. (5 points) For any $w \in \Sigma^*$ we define w^r as $\varepsilon^r = \varepsilon$ and $(wa)^r = aw^r$ for all $w \in \Sigma^*, a \in \Sigma$. That is, w^r is w written backwards. For language L we define L^r as $\{w^r \mid w \in L\}$. Prove that for any regular language L, the language L^r must be regular.

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Solutions for HW#3

1. $M = (\{\{s\},\{s,t\},\{s,t,u\},\{s,t,u,v\}\},\{a,b\},\delta,\{s\},\{\{s,t,u,v\}\})$ where

δ	a	b
$\{s\}$	$\{s,t\}$	$\{s\}$
$\{s,t\}$	$\{s,t,u\}$	$\{s\}$
$\{s,t,u\}$	$\{s,t,u,v\}$	$\{s\}$
$\{s,t,u,v\}$	$\{s,t,u,v\}$	$\{s\}$

2. ***a*** $((0+1)(0+1)(0+1))^*$

b $(0+1)^* 000 (0+1)^*$

c $(1^* 011^* 011^*)^*$

3. ***a*** $L_0 = \{0,1\}^*$ is regular because it is accepted by $M = (\{q_0\},\{0,1\},\delta,q_0,\{q_0\})$ where $\delta(q_0,0) = \delta(q_0,1) = q_0$. It was shown in class and in the text that $L_1 = \{0^n 1^n \mid n \ge 0\}$ is not regular. But $L_0 \cup L_1 = \{0,1\}^*$, which is regular.

b $L_0 = \varnothing$ is regular because it is accepted by $M = (\{q_0\},\{0,1\},\delta,q_0,\varnothing)$ where $\delta(q_0,0) = \delta(q_0,1) = q_0$. It was shown in class and in the text that $L_1 = \{0^n 1^n \mid n \ge 0\}$ is not regular. But $L_0 \cup L_1 = \{0^n 1^n \mid n \ge 0\}$, which is not regular.

4. Assume that $\{a^i b^j c^{2j} \mid i \ge 0, j \ge 0\}$ is regular, and let *k* be as assured by the Pumping Lemma for regular languages. $b^k c^{2k} \in \{a^i b^j c^{2j} \mid i \ge 0, j \ge 0\}$. Then $b^k c^{2k}$ can be written as *uvw* where $1 \le |uv| \le k$ and $|v| \ge 1$, and *v* is a nonempty sequence of *b*'s. But $uv^0 w$ has 2*k* *c*'s, and fewer than *k* *b*'s. The Pumping Lemma says that $uv^0 w \in \{a^i b^j c^{2j} \mid i \ge 0, j \ge 0\}$, which yields a contradiction.

5. If *L* is regular, then there must be DFA $M = (Q,\Sigma,\delta,q_0,F)$ such that $L = L(M)$. That is, $w \in L$ if and only if $\hat{\delta}(q_0,w) \in F$. We design NFA M^* with ε-transitions which starts (nondeterministically) in every state of *F* and which ends in q_0 if and only if input string *w* belongs to *L*. M^* has an ε-transition from its start state to every $q \in F$, and its transition function from states of *Q* has the same edges as δ though with the directions

reversed on the edges. That is, $M^* = \left(Q \cup \left\{q^*\right\}, \Sigma, \delta^*, q^*, \{q_0\}\right)$. $\delta^*(q,a) = \{p\}$ if and only if $\delta(p,a) = q$ and $\delta^*\left(q^*, \varepsilon\right) = F$.