1. (5 points) Do PROBLEM 1 of HOMEWORK 2 on pg. 302 of our text.

2. (3 points) Give regular expressions describing the following languages over \( \{0,1\}^* \):
   a) The set of all strings of lengths divisible by 3.
   b) The set of all strings containing 000.
   c) The set of all strings with an even number of 0's and every 0 is followed by at least one 1.

3. (6 points) a) Give examples of languages \( L_0 \) and \( L_1 \) such that \( 0 \in L_0 \) and \( L_0 \cup L_1 \) are regular but \( L_1 \) is not regular.
   b) Give examples of languages \( L_0 \) and \( L_1 \) such that \( L_0 \) is regular but \( L_1 \) and \( L_0 \cup L_1 \) are not regular.

4. (8 points) Prove that \( \{a^ib^jc^2j \mid i \geq 0, j \geq 0\} \) is not regular.

5. (5 points) For any \( w \in \Sigma^* \) we define \( w' \) as \( \varepsilon' = \varepsilon \) and \( (wa)^r = aw' \) for all \( w \in \Sigma^* \), \( a \in \Sigma \). That is, \( w' \) is \( w \) written backwards. For language \( L \) we define \( L' \) as \( \{w' \mid w \in L\} \). Prove that for any regular language \( L \), the language \( L' \) must be regular.
1. \[ M = \left( \{s\}, \{s,t\}, \{s,t,u\}, \{s,t,u,v\} \right), \{a,b\}, \{s\}, \{\{s,t,u,v\}\} \) where 
\[ \delta \]
\[
\begin{array}{c|c|c}
\text{a} & \text{b} \\
\hline
\{s\} & \{s,t\} & \{s\} \\
\{s,t\} & \{s,t,u\} & \{s\} \\
\{s,t,u\} & \{s,t,u,v\} & \{s\} \\
\{s,t,u,v\} & \{s,t,u,v\} & \{s\}
\end{array}
\]

2. \[ a \left( (0+1)(0+1)(0+1) \right)^* \]
\[ b \left( 0+1 \right)^* 000 \left( 0+1 \right)^* \]
\[ c \left( 1011011 \right)^* \]

3. \[ a \ L_0 = \{0,1\}^\ast \] is regular because it is accepted by \[ M = \left( \{q_0\}, \{0,1\}, \delta, q_0, \{q_0\} \right) \] where 
\[ \delta(q_0,0) = \delta(q_0,1) = q_0. \] It was shown in class and in the text that \[ L_4 = \{0^n1^n \mid n \geq 0 \} \] is not regular. But \[ L_0 \cup L_4 = \{0,1\}^\ast, \] which is regular.

\[ b \ L_0 = \emptyset \] is regular because it is accepted by \[ M = \left( \{q_0\}, \{0,1\}, \delta, q_0, \emptyset \right) \] where 
\[ \delta(q_0,0) = \delta(q_0,1) = q_0. \] It was shown in class and in the text that \[ L_4 = \{0^n1^n \mid n \geq 0 \} \] is not regular. But \[ L_0 \cup L_4 = \{0^n1^n \mid n \geq 0 \}, \] which is not regular.

4. Assume that \[ \{a^ib^jc^{2j} \mid i \geq 0, j \geq 0 \} \] is regular, and let \( k \) be as assured by the Pumping Lemma for regular languages. \[ b^kc^{2k} \in \{a^ib^jc^{2j} \mid i \geq 0, j \geq 0 \}. \] Then \[ b^kc^{2k} \] can be written as \( uvw \) where \( 1 \leq |uv| \leq k \) and \( |v| \geq 1 \), and \( v \) is a nonempty sequence of \( b \)'s. But \( uv^0w \) has \( 2k \) \( c \)'s, and fewer than \( k \) \( b \)'s. The Pumping Lemma says that \[ uv^0w \in \{a^ib^jc^{2j} \mid i \geq 0, j \geq 0 \}, \] which yields a contradiction.

5. If \( L \) is regular, then there must be DFA \( M = (Q, \Sigma, \delta, q_0, F) \) such that \( L = L(M) \). That is, \( w \in L \) if and only if \( \hat{\delta}(q_0, w) \in F \). We design NFA \( M^\ast \) with \( \varepsilon \)-transitions which starts (nondeterministically) in every state of \( F \) and which ends in \( q_0 \) if and only if input string \( w \) belongs to \( L. \) \( M^\ast \) has an \( \varepsilon \)-transition from its start state to every \( q \in F \), and its transition function from states of \( Q \) has the same edges as \( \delta \) though with the directions
reversed on the edges. That is, $M^* = \left( Q \cup \{q^*\}, \Sigma, \delta^*, q^*, \{q_0\} \right)$. $\delta^*(q, a) = \{p\}$ if and only if $\delta(p, a) = q$ and $\delta^*(q^*, \varepsilon) = F$. 