

# CS3133

## HW#2

DUE: Tuesday, September 6

1. (8 points) Prove that the strings in **any** regular set of binary strings with either zero occurrences of 1 or at least two occurrences of 1 is regular. That is, if  $L$  is a regular language of binary strings, then upon removing from  $L$  all strings with exactly one occurrence of 1 we are left with a regular language.

2. (8 points) Prove that for any regular language  $L$  there is an NFA  $N = (Q, \Sigma, \Delta, S, F)$  such that  $L(N) = L$  and  $|S| = |F| = 42$ .

3. (6 points) Describe an NFA to accept the set of all binary strings  $z$  such that either every odd position of  $z$  is a 1 or  $z$  contains at least two 0s.

4 (14 points) Prove or give a counterexample to each of the following:

**CONJECTURE A:** For any NFA without  $\varepsilon$ -transitions  $N = (Q, \Sigma, \Delta, S, F)$ , if  $p, q \in Q - (F \cup S)$ ,  $p \neq q$  with  $\Delta(p, a) = \Delta(q, a)$  and  $\bigcup_{p \in \Delta(r, a)} r = \bigcup_{q \in \Delta(r, a)} r$  for all  $a \in \Sigma$ , then an equivalent NFA is obtained by “lumping”  $p$  and  $q$  into a new state  $pq$ . That is, if  $N^* = (Q - \{p, q\} \cup \{pq\}, \Sigma, \Delta^*, S, F)$  where  $\Delta^*$  is obtained from  $\Delta$  by replacing each instance of  $p$  or  $q$  with the new state  $pq$ , then  $L(N) = L(N^*)$ .

**CONJECTURE B:** For any NFA  $N = (Q, \Sigma, \Delta, S, F)$ , the language accepted by  $(Q, \Sigma, \Delta, F, S)$  is the complement of  $L(N)$ .

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Solutions to HW#2

1. The DFA  $(\{q, r, s\}, \{0, 1\}, \delta, q, \{r\})$ , with

$\delta$	<b>0</b>	<b>1</b>
<b>q</b>	q	r
<b>r</b>	r	s
<b>s</b>	s	s

accepts the set of all binary strings with exactly one occurrence of 1. By the closure properties of regular languages, its complement must also be regular. Again by the closure properties of regular languages, the intersection of this language with  $L$  must also be regular.

2. If  $L$  is regular, then it is accepted by NFA  $N^* = (Q^*, \Sigma, \Delta^*, S^*, F^*)$ . It is also accepted by  $N^* = (Q \cup \{p_1, \dots, p_{42}\} \cup \{q_1, \dots, q_{42}\}, \Sigma, \Delta^*, \{p_1, \dots, p_{42}\}, \{q_1, \dots, q_{42}\})$  such that  $Q \cap \{q_1, \dots, q_{42}\} = \emptyset$  and  $Q \cap \{p_1, \dots, p_{42}\} = \emptyset$  and  $\Delta^*$  is identical to  $\Delta$  except that it has an  $\varepsilon$ -transition from each state of  $\{p_1, \dots, p_{42}\}$  to each state of  $S^*$  and an  $\varepsilon$ -transition from each state of  $F^*$  to each state of  $\{q_1, \dots, q_{42}\}$ .

3.  $q_1$  and  $q_2$  are the states of a machine to accept a language in which every odd position is a 1, and  $p_1$ ,  $p_2$  and  $p_3$  are the states of a machine to accept all strings containing at least two 0s. The union of the languages these machines accept is accepted by the NFA  $N = (\{s, q_1, q_2, p_1, p_2, p_3\}, \{0, 1\}, \Delta, \{s\}, \{q_1, q_2, p_3\})$ , where

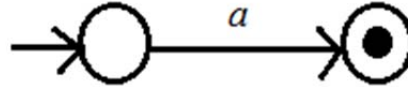
$\Delta$	0	1	$\varepsilon$
s	$\emptyset$	$\emptyset$	$\{q_1, p_1\}$
$q_1$	$\emptyset$	$\{q_2\}$	$\emptyset$
$q_2$	$\{q_1\}$	$\{q_1\}$	$\emptyset$
$p_1$	$\{p_2\}$	$\{p_1\}$	$\emptyset$
$p_2$	$\{p_3\}$	$\{p_2\}$	$\emptyset$
$p_3$	$\{p_3\}$	$\{p_3\}$	$\emptyset$

4. We prove **CONJECTURE A** by showing (by induction on  $|z|$ ) that for any string  $z \in \Sigma^*$ ,  $\hat{\Delta}(S, z) = \hat{\Delta}^*(S, z)$  for all  $S \subseteq Q - \{p, q\}$  and  $(q \in \hat{\Delta}(S, z)) \leftrightarrow (p \in \hat{\Delta}(S, z)) \leftrightarrow (pq \in \hat{\Delta}^*(S, z))$ . It thus follows that  $(\hat{\Delta}(S, z) \cap F = \emptyset) \leftrightarrow (\hat{\Delta}^*(S, z) \cap F = \emptyset)$ , implying that  $L(N) = L(N^*)$ .

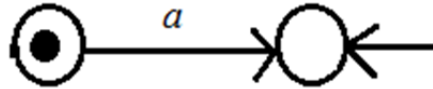
Since  $N$  and  $N^*$  don't have  $\varepsilon$ -transitions, it follows that  $\hat{\Delta}(S, \varepsilon) = \hat{\Delta}^*(S, \varepsilon) = S$ . As an induction hypothesis, assume that for any string  $z \in \Sigma^*$ ,  $\hat{\Delta}(S, z) = \hat{\Delta}^*(S, z)$  for all

$S \subseteq Q - \{p, q\}$  and  $(q \in \hat{\Delta}(S, z)) \leftrightarrow (p \in \hat{\Delta}(S, z)) \leftrightarrow (pq \in \hat{\Delta}^*(S, z))$ . Choose any  $a \in \Sigma$ .  
 $\hat{\Delta}(S, za) = \Delta(\hat{\Delta}(S, z), a) = \Delta(\hat{\Delta}^*(S, z), a) = \hat{\Delta}^*(S, za)$  for all  $S \subseteq Q - \{p, q\}$ , and  
 $(q \in \hat{\Delta}(S, za)) \leftrightarrow (q \in \Delta(\hat{\Delta}(S, z), a)) \leftrightarrow (pq \in \Delta^*(\hat{\Delta}^*(S, z), a)) \leftrightarrow (pq \in \hat{\Delta}^*(S, za))$ , with a  
 symmetric argument for  $p$ .

**CONJECTURE B** is false. The language accepted by



is  $\{a\}$ , but the language accepted by



is  $\emptyset$ .