

CS3133

HW#2

DUE: Tuesday, September 7

1. (8 points) Describe an NFA to accept the language of all nonempty strings over $\{a,b,c\}$ such that the final character does not appear anywhere else in the string. So, $abac$ and b are in the language but ε , $abacc$ and $abca$ are not in the language.
2. (6 points) Show that the set of all binary strings such that the number of occurrences of 01 is equal to the number of occurrences of 10 is regular. Note that 101 belongs to our language but 1010 does not.
3. (8 points) Prove or give a counterexample to the following:
CONJECTURE: A language L is regular if and only if there is an NFA $N = (Q, \Sigma, \Delta, S, F)$ such that $L = L(N)$ and $|S| = |F| = 1$.
4. (12 points) String x is a *proper prefix* of string y if there exists string $z \neq \varepsilon$ such that $xz=y$. Show that for any regular language L , the following two languages must each be regular.
 - a $\text{NOPREFIX}(L) = \{w \in L \mid \text{no proper prefix of } w \text{ is a member of } L\}$.
 - a $\text{NOEXTEND}(L) = \{w \in L \mid w \text{ is not a proper prefix of any member of } L\}$.

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Solutions to HW#2

1. The tricky part of this question is to guess when the symbol being read is the last symbol of the string. States such as q_a and q_{ab} "mean" that an a has been read and that an a and a b have been read, respectively. State q_f "means" that the last symbol read just appeared for the first time in the string. So q_f is the only final state. The language is accepted by $N = (\{s, q_a, q_b, q_c, q_{ab}, q_{ac}, q_{bc}, q_f\}, \{a, b, c\}, \Delta, \{s\}, \{q_f\})$ where

Δ	a	b	c
s	$\{q_a, q_f\}$	$\{q_b, q_f\}$	$\{q_c, q_f\}$
q_a	$\{q_a\}$	$\{q_{ab}, q_f\}$	$\{q_{ac}, q_f\}$
q_b	$\{q_{ab}, q_f\}$	$\{q_b\}$	$\{q_{bc}, q_f\}$
q_c	$\{q_{ac}, q_f\}$	$\{q_{bc}, q_f\}$	$\{q_c\}$
q_{ab}	$\{q_{ab}\}$	$\{q_{ab}\}$	$\{q_f\}$
q_{ac}	$\{q_{ac}\}$	$\{q_f\}$	$\{q_{ac}\}$
q_{bc}	$\{q_f\}$	$\{q_{bc}\}$	$\{q_{bc}\}$
q_f	\emptyset	\emptyset	\emptyset

2. The language is accepted by the DFA $M = (\{q_0, q_1, q_2, q_3, q_4\}, \{0, 1\}, \delta, q_0, \{q_0, q_1, q_3\})$ where

δ	0	1
q_0	q_1	q_3
q_1	q_1	q_2
q_2	q_1	q_2
q_3	q_4	q_3
q_4	q_4	q_3

3. The CONJECTURE is true. Language L is regular if and only if there exists $N = (Q, \Sigma, \Delta, S, F)$ such that $L = L(N)$. In order to satisfy the conditions $|S| = |F| = 1$, we construct $N' = (Q \cup \{q_{start}, q_{accept}\}, \Sigma, \Delta', \{q_{start}\}, \{q_{accept}\})$ where $q_{start}, q_{accept} \notin Q$,

$$\Delta'(q, a) = \begin{cases} \Delta(q, a), & \text{for all } q \in Q, a \in \Sigma \\ \emptyset, & \text{for } q \in \{q_{start}, q_{accept}\} \end{cases} \quad \text{for all } a \in \Sigma$$

and $\Delta'(q_{start}, \varepsilon) = S$ and

$$\Delta'(q, \varepsilon) = \begin{cases} \Delta(q, \varepsilon), & \text{for all } q \in Q - F \\ \Delta(q, \varepsilon) \cup \{q_{accept}\}, & \text{for all } q \in F \end{cases} .$$

4. We assume that regular language L is accepted by DFA $M = (Q, \Sigma, \delta, q_0, F)$.

a For each $x \in L(M)$, we don't want to accept any string $w \in L(M)$ with x as a proper prefix. Since M is a DFA and every string in Σ^* corresponds to a unique path through Q , this is accomplished by converting M to an NFA $M = (Q, \Sigma, \Delta, \{q_0\}, F)$ with

$$\Delta(q, a) = \begin{cases} \{\delta(q, a)\}, & \text{for all } q \in Q - F \\ \emptyset, & \text{for all } q \in F \end{cases} .$$

We just removed all edges (transitions) out of each final state of M . So $L(N) = \text{NOPREFIX}(L)$.

b Considering M to be a graph, we define a set of $|Q| \times |Q|$ boolean variables x_{ij} ,

$1 \leq i, j \leq |Q|$, to be **true** if there is a nontrivial path from q_i to q_j in M and **false** otherwise (a path is *nontrivial* if it consists of at least one edge/transition). The values of x_{ij} can be computed using the Floyd-Warshall algorithm. $\text{NOEXTEND}(L)$ is accepted by a DFA

$M^* = (Q, \Sigma, \delta, q_0, F^*)$ where $F^* = \{q_i \in F \mid \neg(\exists q_j \in F) x_{ij} = \text{true}\}$. That is, to obtain F^* we remove from F any state q_i such that there is a nontrivial path from q_i to some state of F .