

# CS3133

## HW#2

DUE: Monday, September 8

1. (6 points) For languages  $A$  and  $B$ , we define

$$A \parallel B = \{a_1 \dots a_{2k} \mid a_1 a_3 \dots a_{2k-1} \in A \wedge a_2 a_4 \dots a_{2k} \in B\}.$$

For example, if  $0110 \in A$  and  $1101 \in B$ , then  $01111001$  must belong to  $A \parallel B$ . Prove that for any regular languages  $A$  and  $B$ , language  $A \parallel B$  must be regular.

2. (8 points) **a** Motivated by the discussion of  $\varepsilon$ -transitions on pages 36 and 37 of our text, we extend the definition of the transition function  $\Delta: Q \times \Sigma \rightarrow 2^Q$  to allow a transition which does not consume input,  $\Delta_\varepsilon: Q \times (\Sigma \cup \{\varepsilon\}) \rightarrow 2^Q$ . Following the suggestion of Miscellaneous Exercise 10 on page 318 of our text, we define the  $\varepsilon$ -closure  $C_\varepsilon(A)$  of set  $A \subseteq Q$  to be the set of all states reachable from some state in  $A$  under a (possibly empty) sequence of  $\varepsilon$ -transitions.

**a** Define an extended transition function  $\widehat{\Delta}_\varepsilon: 2^Q \times \Sigma^* \rightarrow 2^Q$ . Note that  $\widehat{\Delta}_\varepsilon$  is not necessarily the same as  $\widehat{\Delta}$ , since for any  $A \subseteq Q$  it must be the case that  $\widehat{\Delta}(A, \varepsilon) = A$  but  $\widehat{\Delta}_\varepsilon(A, \varepsilon)$  is not necessarily equal to  $A$ .

**b** Define the language accepted by NFA  $N$  with  $\varepsilon$ -transitions.

**c** What can you say about  $L(M)$  if  $C_\varepsilon(\{S\}) \cap F \neq \emptyset$ ?

3. (5 points) Prove that the language  $\{0,1\}^* 1\{0,1\}\{0,1\}\{0,1\}1\{0,1\}^*$  is regular. This is the set of binary strings which contain two 1's with three bits between them must be regular. For example, 111 does not belong to the language, nor does 001011000001. But 110101000 does belong.

4. (6 points) **a** Construct a three state NFA to accept the language  $0\{0\}^*1\{1\}^*$ .

**b** Construct a DFA to accept the language  $0\{0\}^*1\{1\}^*$ .

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## Solutions to HW#2

1. If  $A$  and  $B$  are regular, then there are DFAs  $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$  and

$M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$  to accept them. An idea behind designing an acceptor for  $A||B$  is to alternate transitions between  $M_1$  and  $M_2$ . We construct DFA

$$M_{||2} = (Q_1 \times Q_2 \times \{\leftarrow, \rightarrow\}, \Sigma, \delta_{||2}, \{(s_1, s_2, \leftarrow)\}, F_{||2})$$

where the idea behind state  $(q_i, q_j, \leftarrow)$  is that the odd characters of the consumed input would send  $M_1$  to state  $q_i$  and the even characters of the consumed input would send  $M_2$  to state  $q_j$  and  $\leftarrow$  means that the next input symbol will cause a simulated transition in  $M_1$ , while  $\rightarrow$  means that the next input symbol will cause a simulated transition in  $M_2$ .

So  $\delta_{||2}((q_i, q_j, \leftarrow), a) = (\delta_1(q_i, a), q_j, \rightarrow)$  and  $\delta_{||2}((q_i, q_j, \rightarrow), a) = (q_i, \delta_2(q_j, a), \leftarrow)$

for all  $q_i \in Q_1, q_j \in Q_2, a \in \Sigma$ . Finally,  $F_{||2} = \{(q_i, q_j, \leftarrow) \mid q_i \in F_1 \wedge q_j \in F_2\}$ .

2. **a** For each  $A \subseteq Q$ ,

- $\widehat{\Delta}_\varepsilon(A, \varepsilon) = C_\varepsilon(A)$

- $\widehat{\Delta}_\varepsilon(A, xa) = \bigcup_{q \in \widehat{\Delta}_\varepsilon(A, x)} \Delta_\varepsilon(q, a)$

**b** We say that  $N$  accepts  $x$  if  $\widehat{\Delta}_\varepsilon(S, x) \cap F \neq \emptyset$ , and  $L(N) = \{x \in \Sigma^* \mid N \text{ accepts } x\}$ .

**c** Since the condition states that there is an  $\varepsilon$ -transition from  $S$  to a final state, we can conclude that  $\varepsilon \in L(N)$ .

3. The idea behind an NFA to accept the language is that, when reading a 1, one guesses whether or not there's another 1 four bits ahead. If not, we can stay in state  $q_0$ ; if there is a 1 four bits ahead, we can jump to state  $q_1$ . An NFA to accept the language is

$N = (\{q_0, q_1, q_2, q_3, q_4\}, \{0, 1\}, \Delta, \{q_0\}, \{q_4\})$  where

$\Delta$	0	1
$q_0$	$\{q_0\}$	$\{q_0, q_1\}$
$q_1$	$\{q_2\}$	$\{q_2\}$
$q_2$	$\{q_3\}$	$\{q_3\}$
$q_3$	$\emptyset$	$\{q_4\}$
$q_4$	$\{q_4\}$	$\{q_4\}$

4. **a** The machine  $(\{q_0, q_1, q_2\}, \{0, 1\}, \Delta, \{q_0\}, \{q_2\})$  consumes 0's until it guesses that it sees the last one, and then consumes 1's until it guesses that it sees the last one.

$\Delta$	0	1
$q_0$	$\{q_0, q_1\}$	$\emptyset$
$q_1$	$\emptyset$	$\{q_1, q_2\}$
$q_2$	$\emptyset$	$\emptyset$

**b** Converting the NFA from 4 **a** to a DFA and eliminating unreachable states yields

$(\{\{q_0\}, \{q_0, q_1\}, \{q_1, q_2\}, \emptyset\}, \{0, 1\}, \delta, \{q_0\}, \{q_1, q_2\})$  where

$\delta$	0	1
$\{q_0\}$	$\{q_0, q_1\}$	$\emptyset$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_1, q_2\}$
$\{q_1, q_2\}$	$\emptyset$	$\{q_1, q_2\}$
$\emptyset$	$\emptyset$	$\emptyset$