CS3133

HW#2

DUE: Tuesday, September 6

1. (3 points) Assume that L is a regular language. Show that the set of prefixes of strings in L is also regular. That is, $\{u \mid \exists v(uv \in L)\}$ is regular.

2. (3 points) Show that the set of all strings z over $\{0,1\}$ such that $|z| \geq 3$ and the third character from the end of z is a 0 is a regular language. For example, 001100 does not belong to the language, but 001000 does.

3. (3 points) Give a regular expression for the language $L \subseteq \{0,1\}^*$ such that L contains all strings with an even number of 0's and every 0 is followed by at least one 1. For example, $\{\varepsilon, 011101, 1, 010110101\} \subseteq L$ but $010 \notin L$.

4. (9 points) Prove that for any regular language L, there is an NFA $M = (Q, \Sigma, \delta, s, F)$ such that $L = L(M)$ and $|F| = 1$. Don't just give a construction; prove that L belongs to the language described by or accepted by your construction.

CS3133

Solutions for HW#2

1. If L is regular, then it is accepted by a DFA $M=(Q,\Sigma,\delta,s,F)$. Also, u is a prefix of $uv\in L$ if and only if there is a path in M from s to $\hat{\delta}(s,u)$ and there is a path in M from $\hat{\delta}(s,u)$ to a state in F. Let $S\subseteq Q$ be the states such that for each $q\in S$, there is a path in M from s to q **and** there is a path in M from q to a state of F. Language $\{u \mid \exists v(uv\in L)\}$ is accepted by DFA $(Q,\Sigma,\delta,s,F\cup S)$, and hence it is regular.

2. The idea is to design an NFA M which guesses that a symbol being read is the third from the end, and then verifies that it's a 0 (go to q_1) and that exactly two more symbols follow. $M=(\{s,q_1,q_2,q_3\},\{0,1\},\delta,s,\{q_3\})$, where

δ	0	1
s	$\{s,q_1\}$	$\{s\}$
q_1	$\{q_2\}$	$\{q_2\}$
q_2	$\{q_3\}$	$\{q_3\}$
q_3	$\varnothing$	$\varnothing$

3. $1^*(011^*011^*)^*$

4. If L is regular, then there is an NFA $M*=(Q,\Sigma,\Delta^*,S,F)$ without ε-transitions such that $L=L(M^*)$. We construct $M=(Q\cup\{f\},\Sigma,\Delta,S^*,\{f\})$ where

$$S^*=\begin{cases} S, \text{if } S\cap F=\varnothing \\ S\cup\{f\}, \text{otherwise}\end{cases}.$$

For any transition to a final state of M^*, we add to M an "equivalent" transition to the new, unique final state f. Also, M doesn't permit any transitions from f. That is, for all $A\subseteq Q\cup\{f\}, a\in\Sigma$,

$$\Delta(A,a)=\begin{cases}\Delta^*(A,a), \text{ if } \Delta^*(A,a)\cap F=\varnothing \\ \Delta^*(A,a)\cup\{f\}, \text{ otherwise}\end{cases}$$

and $\Delta(\{f\},a)=\varnothing$.

We use induction to show that for any $w\in\Sigma^*$.

$$\hat{\Delta}(S,w)=\begin{cases}\widehat{\Delta^*}(S,w), \text{if } \widehat{\Delta^*}(S,w)\cap F=\varnothing \\ \widehat{\Delta^*}(S,w)\cup\{f\}, \text{otherwise}\end{cases}.$$

That is, if we ignore *f*, then $\hat{\Delta}$ and $\widehat{\Delta^*}$ do exactly the same thing.
As a basis , $w=\varepsilon$, we first note that from the definition of S^*,

$$\hat{\Delta}(S,\varepsilon)=\begin{cases} S,\text{ if } S\cap F=\varnothing \\ S\cup\{f\},\text{ otherwise}\end{cases}.$$

For an induction hypothesis, we assume that

$$\hat{\Delta}(S,w)=\begin{cases} \widehat{\Delta^*}(S,w),\text{ if } \widehat{\Delta^*}(S,w)\cap F=\varnothing \\ \widehat{\Delta^*}(S,w)\cup\{f\},\text{ otherwise}\end{cases}.$$

For any $a\in\Sigma$, even if $f\in\hat{\Delta}(S,w)$, there are no transitions out of *f*, $\Delta(\{f\},a)=\varnothing$. Hence, $\hat{\Delta}(S,wa)=\Delta\left(\hat{\Delta}(S,w),a\right)$ is equal to $\widehat{\Delta^*}(S,w)$ if $\widehat{\Delta^*}(S,w)\cap F=\varnothing$ and is equal to $\widehat{\Delta^*}(S,w)\cup\{f\}$ if $\widehat{\Delta^*}(S,w)\cap F\neq\varnothing$, which is what we are trying to prove.