CS3133
HW#1 SOLUTIONS

1. $L_{\overline{001}}$ is accepted by the DFA $A_{\overline{001}} = \left(\{q_0, q_1, q_2, q_3\}, \{0,1\}, d_{\overline{001}}, \{q_0, q_1, q_2\}\right)$, where

$d_{\overline{001}}$	0	1
q_0	q_1	q_0
q_1	q_2	q_0
q_2	q_2	q_3
q_3	q_3	q_3

and L_{001} is accepted by the DFA $A_{001} = \left(\{q_0, q_1, q_2, q_3\}, \{0,1\}, d_{\overline{001}}, \{q_3\}\right)$.

2. L is accepted by the DFA $A = \left(\{q_0, q_1, q_2, q_3, q_4\}, \{0,1,2,3\}, d, \{q_0\}\right)$, where

d	0	1	2	3
q_0	q_0	q_1	q_2	q_3
q_1	q_1	q_2	q_3	q_4
q_2	q_2	q_3	q_4	q_0
q_3	q_3	q_4	q_0	q_1
q_4	q_4	q_0	q_1	q_2

3. a) **BASIS**: $\hat{d}\left(q, a^0\right) = \hat{d}\left(q, e\right) = q$ since we are in state q and read no inputs, and hence we don't change state.

INDUCTION: Assume that for all states $q \in Q$, $\hat{d}\left(q, a^n\right) = q$. By the definition of the extended transition function, $\hat{d}\left(q, a^{n+!}\right) = \hat{d}\left(q, a^n a\right) = d\left(\hat{d}\left(q, a^n\right), a\right)$. By the INDUCTION HYPOTHESIS, this is $d\left(q, a\right) = q$.

b) If $q_0 \in F$, then $\hat{d}\left(q_0, a^n\right) = q_0 \in F$ and $\{a\}^* \subseteq L(A)$. Likewise, if $q_0 \notin F$, then for all n, $\hat{d}\left(q_0, a^n\right) = q_0 \notin F$ and $\{a\}^* \cap L(A) = \emptyset$.