CS3133

HW#1

DUE: Tuesday, August 30

1. (3 points) Let $L_0 = \{000\}^*$, $L_1 = \{0,1\}\{0\}$ and $L_2 = (L_1)^*$.

   a Describe $L_0 \cap L_2$.

   b Is $(L_0\{\varepsilon\}) \cap L_1 \subseteq L_2$?

   c Describe $2^{L_0 \cap L_1}$, where the power set of any language L is $2^L \stackrel{def}{=} \{S \mid S \subseteq L\}$.

2. (8 points) For each of the following statements, tell whether or not it is true for **all** languages L_0 and L_1. Justify your answers.

   a $2^{L_0} \cup 2^{L_1} = 2^{L_0 \cup L_1}$.

   b $2^{L_0} \cup 2^{L_1} \subseteq 2^{L_0 \cup L_1}$.

   c $|L_0 L_1| = |L_0| * |L_1|$.

   d $L_0^* = L_0 L_0^*$.

3. (14 points) For both answers below, do not worry about using a minimal number of states.

a Consider the language L consisting of binary strings of length at least 2 such that the next to the last symbol read is a 0. So $0100 \in L$ and $0000 \in L$, but $0110 \notin L$ and $0 \notin L$. Show that L is regular.

b Let n be a non-negative integer and let L_n be the set of binary strings of length at least n and such that the $n^{\underline{th}}$ symbol from the end is a 0. So L from part ***a*** is L_2, and $0100 \in L_4$ and $0000 \in L_1$ and $101111 \in L_5$, but $0110 \notin L_3$ and $0 \notin L_8$. Show that L_n is regular.

4 (6 points) Show that the set of all binary strings which do not contain 110 as a substring is regular.

5 (10 points) Let L be a language over Σ, $|\Sigma| > 1$, such that a longest string in L has length n. Show that there is a DFA $M = (Q, \Sigma, \delta, s, F)$ such that $L(M) = L$ and $|Q| \le |\Sigma|^{n+1}$.

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Solutions to HW#1

1. ***a*** $L_0 \cap L_2 = \{000000\}^*$. That is, $L_0 \cap L_2$ is the set of all strings of 0's of length divisible by 6.
b Yes, because $(L_0\{\varepsilon\}) \bigcap L_1 = \varnothing \bigcap L_1 = \varnothing$.
c $L_0 \cap L_1 = \varnothing$, so $2^{L_0 \cap L_1} = \{\varnothing\}$. Note that $\{\varnothing\} \neq \varnothing$.

2. ***a*** $2^{L_0} \cup 2^{L_1} = 2^{L_0 \cup L_1}$ is sometimes false. If $L_0 = \{0\}$ and $L_1 = \{1\}$, then $2^{L_0} \cup 2^{L_1} = \{\varnothing, \{0\}\} \cup \{\varnothing, \{1\}\} = \{\varnothing, \{0\}, \{1\}\} \neq \{\varnothing, \{0\}, \{1\}, \{0,1\}\} = 2^{\{0,1\}} = 2^{L_0 \cup L_1}$.
b $2^{L_0} \cup 2^{L_1} \subseteq 2^{L_0 \cup L_1}$ is always true. If language $L \in 2^{L_0} \cup 2^{L_1}$, then $L \in 2^{L_0}$ or $L \in 2^{L_1}$, implying that $L \subseteq L_0 \vee L \subseteq L_1$. So $L \subseteq L_0 \cup L_1$, and $L \in 2^{L_0 \cup L_1}$.
c $|L_0 L_1| = |L_0| * |L_1|$ is sometimes false. If $L_0 = L_1 = \{0, 00\}$, then $|L_0 L_1| = |\{00, 000, 0000\}| = 3 \neq 4 = |\{0, 00\}| * |\{0, 00\}| = |L_0| * |L_1|$.
d $L_0^* = L_0 L_0^*$ is true if and only if $\varepsilon \in L_0$. If $\varepsilon \notin L_0$, then $\varepsilon \in L_0^*$ though $\varepsilon \notin L_0 L_0^*$.

3. For each binary string *z* of length at most *n*, there is a state q_z which "means" that the last $|z|$ symbols read were *z*. The final states are the states $\left\{q_{0x} \mid x \in \{0,1\}^{n-1}\right\}$. The initial state is q_ε. For $z \in \{0,1\}^*$, $|z| < n, a \in \{0,1\}$, $\delta(q_z, a) = q_{za}$ and if $z \in \{0,1\}^{n-1}$ and $b \in \{0,1\}$, then $\delta(q_{bz}, a) = q_{za}$.

4. $L_{\overline{110}}$ is accepted by the DFA $A_{\overline{110}} = (\{q_0, q_1, q_2, q_3\}, \{0,1\}, \delta_{\overline{110}}, q_0, \{q_0, q_1, q_2\})$, where

$\delta_{\overline{110}}$	0	1
q_0	q_0	q_1
q_1	q_0	q_2
q_2	q_3	q_2
q_3	q_3	q_3

5. *L* is accepted by a DFA *M* with a state q_z corresponding to every $z \in \Sigma^*$, $|z| \leq n$, plus a state $q_{BlackHole}$. The final states of *M* are the members of $\{q_z \mid z \in L\}$, and the transition function of *M* is

$$\delta(q_z, a) = \begin{cases} q_{za}, \text{ if } |z| < n \\ q_{BlackHole}, \text{ if } |z| = n \end{cases}$$

and $\delta(q_{BlackHole}, a) = q_{BlackHole}$ for all $a \in \Sigma$. The initial state of *M* is q_ε.