CS3133

HW#1

DUE: Tuesday, September 2

1. (8 points) Let $A = \{00, 11\}$ and $B = \{\varepsilon, 1, 01\}$.
   a List the strings in AB.
   b What is the cardinality of $\{\alpha \mid \alpha \in A^* \wedge |\alpha| = 6\}$?
   c List the members of $\{\alpha \mid \alpha \in B^* \wedge |\alpha| \leq 3\}$.
   d List the members of $\{\alpha \mid \alpha \in A^* B^* \wedge |\alpha| \leq 4\}$.

2. (5 points) Let $L_1 = \{000\}^*$, $L_2 = \{0,1\}\{0,1\}\{0,1\}\{0,1\}$ and $L_3 = L_2^*$. Describe $L_1 \cap L_3$. What is the cardinality of $L_1 \cap L_3$?

3. (7 points) Show that the language $L = \left\{ x_1 \ldots x_n \mid x_1 \ldots x_n \in \{0,1,2\}^* \wedge \sum_{1 \leq i \leq n} x_i = 0 \bmod 3 \right\}$ is regular. That is, the set of all strings of 0's, 1's and 2's such that the sum of the digits of the string is equal to 0 mod 3 is a regular language. For example, $1200201 \in L$, $00 \in L$, $\varepsilon \in L$, $110010 \in L$, but $12100 \notin L$ and $2 \notin L$.

4. (5 points) Consider the following languages over the alphabet $\Sigma = \{0,1,2\}$:
   - $L_1 = \left\{ x_1 \ldots x_n \mid x_1 \ldots x_n \in \Sigma^* \wedge \sum_{1 \leq i \leq n} x_i = 0 \bmod 3 \right\}$
   - L_2 is the set of strings that contain a 1.

   Prove that $L_1 \cup L_2$ is regular. For example, $1200201 \in L_1 \cup L_2$, $00 \in L_1 \cup L_2$, $\varepsilon \in L_1 \cup L_2$, $110010 \in L_1 \cup L_2$, $12100 \in L_1 \cup L_2$ but $2 \notin L_1 \cup L_2$.

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Solutions to HW#1

1. ***a*** $\{00,11,001,111,0001,1101\}$

b 8

c $\{\varepsilon,1,01,11,101,011,111\}$

d $\{\varepsilon,1,00,01,11,001,011,101,111,0000,0001,0011,0111,1100,1101,1111,\}$

2. $L_1 \cap L_3 = \{0^{12k} \mid k \geq 0\}$, and it is infinite.

3. L is accepted by the DFA $M = (\{q_0, q_1, q_2\}, \{0,1,2\}, \delta, q_0, \{q_0\})$ where state $q_i, 0 \leq i \leq 2$, "means" that the sum of the characters read so far sums to i modulo 3.

δ	0	1	2
q_0	q_0	q_1	q_2
q_1	q_1	q_2	q_0
q_2	q_2	q_0	q_1

4. $L_1 \cup L_2$ is accepted by the DFA $M = (\{q_0, q_1, q_2, q_3\}, \{0,1,2\}, \delta, q_0, \{q_0, q_3\})$ where state $q_i, 0 \leq i \leq 2$, "means" that a 1 has not yet been read and the sum of the characters read so far sums to i modulo 3, and state q_3 "means" that a 1 has been read.

δ	0	1	2
q_0	q_0	q_3	q_2
q_1	q_1	q_3	q_0
q_2	q_2	q_3	q_1
q_3	q_3	q_3	q_3