CS3133

HW#1

DUE: Tuesday, August 28

1. (5 points) Give a necessary and sufficient condition on language L for L^* to be finite. That is, fill in the ### in the following THEOREM.

   THEOREM: For any language L, L^* is finite if and only if ####.

2. (7 points) Let $L \subseteq \{0,1\}^*$ be the set of all binary strings which contain the substring 011. That is, $111 \notin L$, $0011100 \in L$, $010001 \notin L$ and $1000101 \notin L$. Show that L is regular.

3. (10 points) Prove that any finite language over a finite set Σ is regular. Hint: Consider the DFA to accept $\{0,1,111\}$.

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Solutions to HW#1

1. THEOREM: For any language L, L^* is finite if and only if $L = \varnothing$.

2. L is accepted by $M = \left(\{q, q_0, q_{01}, q_{011}\}, \{0,1\}, \delta, q, \{q_{011}\}\right)$, where state q_0 means the last symbol read was a 0, state q_{01} means the last two symbols read were 01, and state q_{011} means that a 011 has been read (though not necessarily the last three symbols).

δ	0	1
q	q_0	q
q_0	q_0	q_{01}
q_{01}	q_0	q_{011}
q_{011}	q_{011}	q_{011}

3. If $L = \varnothing$ then it is accepted by $M = \left(\{q\}, \Sigma, \delta, q, \varnothing\right)$, where $\delta(q,a) = q$ for all $a \in \Sigma$. Otherwise let n be the length of a longest string appearing in L. We construct an acceptor for L with 2^{n+1} states $Q = \{q_w \mid w \in \Sigma^* \wedge |w| \leq n\} \cup \{q_{blackhole}\}$ and initial state q_ε. The final states of our acceptor are $\{q_w \mid w \in L\}$. The transition function of our acceptor is defined for all $q_w \in Q, a \in \Sigma$ by $\delta(q_{blackhole}, a) = q_{blackhole}$ and

$$\delta(q_w, a) = \begin{cases} \text{if } |w| < n, \text{then } q_{wa} \\ \text{else } q_{blackhole} \end{cases} .$$

As an example in case $L \neq \varnothing$, for $L = \{0, 1, 111\}$ we construct the acceptor with states

$$\{q_\varepsilon, q_0, q_1, q_{00}, q_{01}, q_{10}, q_{11}, q_{000}, q_{001}, q_{010}, q_{011}, q_{100}, q_{101}, q_{110}, q_{111}, q_{blackhole}\},$$

initial state q_ε, and final states $\{q_0, q_1, q_{111}\}$. The transition function is

δ	0	1
q_{ε}	q_0	q_1
q_0	q_{00}	q_{01}
q_1	q_{10}	q_{11}
q_{00}	q_{000}	q_{001}
q_{01}	q_{010}	q_{011}
q_{10}	q_{100}	q_{101}
q_{000}	$q_{blackhole}$	$q_{blackhole}$
q_{001}	$q_{blackhole}$	$q_{blackhole}$
q_{010}	$q_{blackhole}$	$q_{blackhole}$
q_{011}	$q_{blackhole}$	$q_{blackhole}$
q_{100}	$q_{blackhole}$	$q_{blackhole}$
q_{101}	$q_{blackhole}$	$q_{blackhole}$
q_{110}	$q_{blackhole}$	$q_{blackhole}$
q_{111}	$q_{blackhole}$	$q_{blackhole}$
$q_{blackhole}$	$q_{blackhole}$	$q_{blackhole}$
q_{11}	q_{110}	q_{111}