CS2223

Midterm Exam

Name______________________________

Date: November 19, 1999

All documentation permitted

1. (12 points) Given an array T of n numbers, and given integer k, $n \geq k \geq 1$, we seek the k^{th} smallest member of T. Consider the following two algorithms:

ALGORITHM A:

HEAPSORT T

return $T[k]$

ALGORITHM B:

INSERTIONSORT T

return $T[k]$

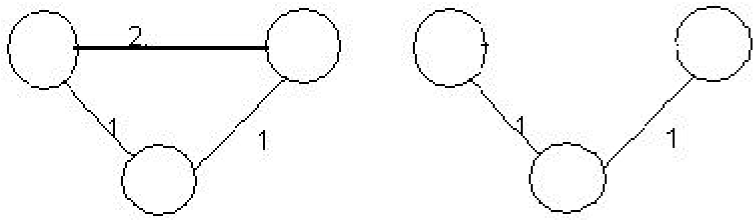
Answer true or false to each of the following three statements:

(a) ALGORITHM A will solve the problem in worst-case time in $O(n + k \lg n)$.

(b) ALGORITHM B will solve the problem in worst-case time in $O(n + k \lg n)$.

(c) ALGORITHM B will solve the problem in best-case time in $O(n + k \lg n)$.

2. (25 points) Noting that the minimum spanning trees of the following two graphs are equal,



Ben makes the following claim:

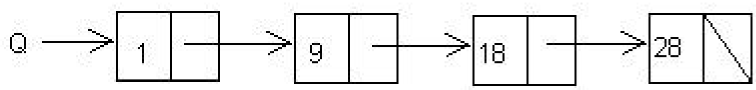
CONJECTURE: For any graph G=<N,A> with function $length : A \rightarrow R^{+}$, let $a \in A$ be an arc of maximum length (every other arc in A is shorter than a). Let H be formed from G by removing a. The minimum spanning tree of G must be equal to the minimum spanning tree of H. That is, we can remove the longest arc in any graph without changing its minimum spanning tree.

Prove or give a counterexample to Ben's Conjecture.

3. (28 points) Suppose that you want to implement the abstract data type Priority Queue using an ordered linked list as a data structure, with standard linked list operations. That is, after executing

construct(Q)
insert(18,Q)
insert(1,Q)
insert(28,Q)
insert(9,Q)

The data structure holding Q would look like



Using Θ–notation, what is the worst-case time to implement the following instructions? (For “insert” and “delete_min”, assume that Q contains n elements.)

construct(Q)

insert(x,Q)

delete_min(Q)

4. (35 points) State whether each of the following claims is true or false, and justify your response.

(a) $\frac{n^2+n}{2} \in O(6n)$

(b) $6n \in O\left(\frac{n^2+n}{2}\right)$

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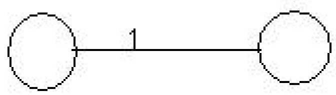
Solutions to Midterm Exam

1. (a) false For k=1, the time constraint is $O(n)$, and HEAPSORT takes worst-case time in $O(n\lg n)$
(b) false For k=n, the time constraint is $O(n\lg n)$, and INSERTIONSORT takes worst-case time in $O(n^2)$.
(c) true The best-case execution time of INSERTIONSORT is $O(n)$, and for any k between 1 and n, the time constraint is at least linear. That is, $n \in O(n + k\lg n)$.

2. The conjecture is false, although it is true for all graphs in which a longest arc belongs to a cycle. To see that it is false, we note that removing the longest arc from G



disconnects the graph (H would not have a spanning tree).

3. construct(Q) - $\Theta(1)$
insert(x,Q) - $\Theta(n)$
delete_min(Q) - $\Theta(1)$

4. (a) false If it were true, there would exist c, n_0 such that $\frac{n^2+n}{2} \le c6n$ for all $n \ge n_0$. Dividing both sides of the inequality by $6n$, this would imply that for all n sufficiently large, $\frac{n}{12} \le \frac{n}{12} + \frac{1}{12} \le c$. But this is impossible. For any c, simply choose n>12c to derive a contradiction.

(b) true We must show that there exist c, n_0 such that $6n \le c\frac{n^2+n}{2}$ for all $n \ge n_0$. Dividing both sides of the inequality by $6n$,

$$1 \le c\frac{n}{12} + \frac{c}{12}$$

Choosing c=12 and n_0=1 yields the desired result.