CS2223 Name_____

Midterm Exam

Date : April 11, 1997

All documentation permitted

1. (30 points) The *median* of a set of n numbers is the $\frac{n+1}{2}$ largest number. For example, the median of (8, 18, 6) is 8. Provide an $O(n^2)$ **upper-bound** on the complexity of finding the median of a list of n numbers.

2. (35 points) An unusual sorting algorithm proceeds by first calling a procedure *C* which *counts* the number of keys less than each key. What is the **exact** (do **not** use *O* or Θ notation) number of times that elements of array *Count* are incremented by *C*. That is, what is the **exact** number of times that statements ✾ or ❍ are executed.

```
procedure C (T : LIST);
    var Count : array [1..n] of integer;
        i,j : 1..n;
    begin
     for i := 1 to n do {
          Count[i] := 0; }
      for i := n downto 2 do {
          for j := i-1 downto 1 do {
            if T [i]< T [j] then
               Count[j] := Count[j] + 1                    ✾
                else
               Count[i] := Count[i] + 1 }}                 ❍
     end; {C}
```

3. (35 points) Given a connected graph $G=(N,A)$ in which the edges all have distinct lengths, prove or give a counterexample to the following

CONJECTURE: Among all paths in G of one edge, the shortest such path must belong to a minimum spanning tree.

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Solutions to Midterm Exam

1. We can use any sorting algorithm with time complexity in $O(n^2)$ to sort the list, and then select the median in time in $O(1)$.

2. $$\sum_{i=2}^{n}\sum_{j=1}^{i-1}1 = \sum_{i=2}^{n}(i-1) = \sum_{i=2}^{n} i - \sum_{i=2}^{n}1 = \frac{n(n+1)}{2} - 1 - (n-1) = \frac{n^2 - n}{2}$$

3. The conjecture is true. A shortest path of one edge is a shortest edge, and Kruskal's algorithm assures us that a shortest edge always belongs to a minimum spanning tree.