CS2223

MIDTERM EXAM

Name______________________________

DATE: November 19, 2001
All documentation permitted

1. (20 points) Suppose that, given a sequence of n constants, $(a_0, a_1, ..., a_{n-1})$, we want to evaluate the polynomial $P(x) = \sum_{0 \le i < n} a_i x^i$ at $x = 42$. Consider the following fragment of code:

> *poly* $\leftarrow$ 0
> **for** *i* $\leftarrow$ *n*-1 **downto** 0 **do**
>   ***poly*** $\leftarrow$ 42 * *poly* + a_i

Assuming that all arithmetic operations can be done in time $O(1)$, what is the time complexity of the above fragment of code? Use Θ-notation and use the simplest function possible.

2. (20 points) Prove or disprove each of the following.

<u>CONJECTURE 1</u>: $2^{n+6} \in O\left(3^n\right)$.

<u>CONJECTURE 2</u>: $3^n \in O\left(2^{n+6}\right)$.

3. (20 points) Let T be an unsorted array of n integers. Give an algorithm to find a pair $x, y \in T$ which minimizes $|x-y|$. If $T = (13, 6, 19, 3, 8)$, then x=6 and y=8 would be a solution. The worst case execution time of your algorithm must be in $O(n \lg n)$.

4. (20 points) Assume that you are given a set T of $n \geq 13$ distinct elements. Show how to find the 13th smallest element of T using, in the worst-case, $n + (12\lceil \lg n \rceil)$ pairwise comparisons.

5. (20 points) Given a connected graph *G* in which the edges all have distinct positive lengths, prove or disprove the following

<u>CONJECTURE</u>: Among all paths in *G* of one edge, the shortest such path must belong to a minimum spanning tree.

CS2223

Solutions to Midterm Exam

1. $\Theta(n)$

2. ***a*** $2^{n+6} \in O\left(3^n\right)$ is true. Choose $n_0 = 1$ and $c = 2^6 = 64$. For all $n \ge 1$,
$2^{n+6} = 64 * 2^n \le 64 * 3^n = c * 3^n$

b $3^n \in O\left(2^{n+6}\right)$ is false. If it were true, then there would exist n_0 and c such that for all $n \ge n_0$, $3^n \le c * 2^{n+6} = 64 * c * 2^n$. Dividing both sides of the inequality by $64 * 2^n$ yields that $\frac{1}{64} * 1.5^n \le c$, which is impossible since $\lim_{n \to \infty} \frac{1}{64} * 1.5^n = \infty$.

3. HEAPSORT(T)
DIFF $\leftarrow \infty$
for $i \leftarrow 2$ **to** n **do**
  if (DIFF > $\left|T[i] - T[i-1]\right|$) **then** INDEX $\leftarrow i$
$x \leftarrow T$[INDEX-1]
$y \leftarrow T$[INDEX]

4. Set up a tournament of the n elements, using n-1 comparisons. Remove the next 12 smallest elements, using $\lceil \lg n \rceil$ comparisons each time.

5. The CONJECTURE is true since the shortest path of one edge is the shortest edge and we know that this path will be added to the minimum spanning tree constructed by Kruskal's Algorithm.