CS2223
MIDTERM EXAM
Name________________

Date: November 18, 2004
All documentation permitted

1 ________

2 ________

3 ________

4 ________

TOTAL ________
1. (25 points) Suppose that $A[1..n]$ is a sorted list of distinct numbers, and $B[1..n]$ is a sorted list of distinct numbers. That is, $A[i] < A[i+1]$ and $B[i] < B[i+1]$ for $1 \leq i < n$. We want to know if the lists are disjoint. That is, we want to know if there are $i, j, 1 \leq i, j \leq n$ such that $A[i] = B[j]$. Show a $O(n)$ upper bound on the complexity of testing if $A$ and $B$ are disjoint.
2. (25 points) Suppose you are given a sorted list $A[1..n]$ of $n > 1$ distinct integers, and a pair of integers $lo$ and $hi$ satisfying $A[1] \leq lo \leq hi \leq A[n]$. Describe an algorithm to return the interval of $A$ whose values fall within $lo$ and $hi$. That is, if your algorithm should return $i$ and $j$, $1 \leq i \leq j \leq n$ satisfying

- $A[k] < lo$ for $k < i$,
- $lo \leq A[k] \leq hi$ for $i \leq k \leq j$, and
- $hi < A[k]$ for $k > j$.

Your algorithm should have a worst-case execution time in $O(\lg n)$. 
3. (25 points) For each of the bottom five rows and for each of the rightmost three columns, check all of the columns which apply. For example, if \( \frac{n^2 - 12n}{4} \in O(65n) \), then check the 3\(^{rd}\) column of the 2\(^{nd}\) row.

<table>
<thead>
<tr>
<th>( f(n) )</th>
<th>( g(n) )</th>
<th>( f(n) \in O(g(n)) )</th>
<th>( g(n) \in O(f(n)) )</th>
<th>( f(n) \in \Theta(g(n)) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{n^2 - 12n}{4} )</td>
<td>65n</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( n\sqrt{n}/2 )</td>
<td>( n\lg n )</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>( 2^n )</td>
<td>( 3^n )</td>
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<td></td>
<td></td>
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<tr>
<td>( 42n^2\sqrt{n} )</td>
<td>( n^2(\sqrt{n} + \ln n) )</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>( n! )</td>
<td>( 4^n )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4. (25 points) Give pseudocode for an algorithm TESTIFHEAP? which will test if array $A[1..n]$ is a (min-)heap using, in the worst-case, $O(n)$ operations. You may assume that $n$ is odd.
CS2223
Solutions to Midterm Exam

1. \text{MERGE}(A[1..n], B[1..n]) \Rightarrow C[1..2n]

   \text{disjoint?} \leftarrow \text{true} \quad \Theta(n)

   \text{for } i \leftarrow 1 \text{ to } 2n-1 \text{ do}
   \hspace{1cm} \text{if } C[i] = C[i+1] \text{ then } \text{disjoint?} \leftarrow \text{false} \quad \Theta(n)

   \text{return disjoint?} \quad O(1)

2. Do a \text{BINARYSEARCH} of \text{lo} in \text{A}, returning either \text{i} such that \text{A[i] = lo} or the index \text{i} of the smallest element of \text{A} greater than \text{lo}.

   Do a \text{BINARYSEARCH} of \text{hi} in \text{A}, returning either \text{j} such that \text{A[j] = hi} or the index \text{j} of the largest element of \text{A} less than \text{hi}.

   \quad O(\lg n)

3.

<table>
<thead>
<tr>
<th>\text{f(n)}</th>
<th>\text{g(n)}</th>
<th>\text{f(n)} \in O(g(n))</th>
<th>\text{g(n)} \in O(f(n))</th>
<th>\text{f(n)} \in \Theta(g(n))</th>
</tr>
</thead>
<tbody>
<tr>
<td>\frac{n^2 - 12n}{4}</td>
<td>65n</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>\frac{n\sqrt{n}}{2}</td>
<td>n\lg n</td>
<td>X</td>
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</tr>
<tr>
<td>\text{2}^n</td>
<td>\text{3}^n</td>
<td>X</td>
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<td>42n^2\sqrt{n}</td>
<td>n^2 \left(\sqrt{n} + \ln n\right)</td>
<td>X</td>
<td>X</td>
<td>X</td>
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<td>4^n</td>
<td></td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>

4. \text{Heap?} \leftarrow \text{true}

   \text{for } i \leftarrow 1 \text{ to } \lfloor n/2 \rfloor \text{ do}
   \hspace{1cm} \text{if } A[i] > A[2i] \text{ or } A[i] > A[2i+1] \text{ then } \text{Heap?} \leftarrow \text{false}

   \text{return heap?}