CS2223

MIDTERM EXAM

Name______________

Date: November 20, 2003
All documentation permitted

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TOTAL	________

1. (25 points) Suppose that you are given an array $A[1..n]$ of numbers, and you seek a maximally distant pair of numbers in A. That is, you seek i and j, $1 \le i, j \le n$ such that for all k and l, $1 \le k, l \le n$,

$$\left|A[i] - A[j]\right| \ge \left|A[k] - A[l]\right|.$$

Show how to solve this problem in worst case time in $O(n)$.

2. (25 points) Suppose that you are given an array $A[1..n]$ of integers, which is *partially sorted* in the sense that there exists an integer m, $1=m=n$, such that $A[1] \leq A[2] \leq ... \leq A[m]$ and $A[m+1] \leq A[m+2] \leq ... \leq A[n]$. However, you do not know the value of m. Show that A can be sorted in worst-case time in $O(n)$.

3. (25 points) Assume we are given as input a binary array $A[1..n]$, that is, $A[i] \in \{0,1\}$ for all $1 \le i \le n$.

a. Prove an upper bound on the complexity of sorting A by showing that it is in $O(n)$.

b . Prove a lower bound on the complexity of sorting A by showing that it is in $\Omega(n)$.

4. (25 points) Prove or give a counterexample to the following.

CONJECTURE: For any connected graph $G = (V, E)$ with $|V| \geq 3$ and one-to-one function $w: E \rightarrow \mathbb{R}^+$ which associates a unique weight with every edge ($e \neq e' \rightarrow w(e) \neq w(e')$), the two edges of minimum weights belong to a minimum spanning tree of G.

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Solutions to Midterm Exam

1. This problem is solved when $A[i]$ is the minimum element of A and $A[j]$ is the maximum element. Both of these values can be computed in linear time.

▸ Set i to index of minimum element
$i \leftarrow 1$
for $k \leftarrow 2$ **to** n **do**
  if $A[k] < A[i]$ **then** $i \leftarrow k$
▸ Set j to index of maximum element
$j \leftarrow 1$
for $k \leftarrow 2$ **to** n **do**
  if $A[k] > A[j]$ **then** $j \leftarrow k$

2. $m \leftarrow 1$
▸ Find m
repeat $m \leftarrow m+1$ **until** $\left(A[m] > A[m+1] \vee m = n-1\right)$
if $m<n$ **then** MERGE($A[1..m]$,$A[m+1..n]$)

3. ***a*** We establish the upper bound by providing a linear time algorithm.
$num0s \leftarrow 0$
▸ count the 0s in A
for $i \leftarrow 1$ **to** n **do**
  if $A[i] = 0$ **then** $num0s \leftarrow num0s + 1$
▸ fill in the 0s, then the 1s in A
for $i \leftarrow 1$ **to** $num0s$ **do**
  $A[i] \leftarrow 0$
for $i \leftarrow num0s+1$ **to** n **do**
  $A[i] \leftarrow 1$

b To establish a linear time lower bound, we note that if an algorithm existed to sort A without examing some element, say $A[i^*]$, then we could change $A[i^*]$, leaving the other n-1 elements of A unchanged, and the algorithm would have to yield the same result, which is now incorrect.

4. The CONJECTURE is true. Neither edge can introduce a cycle (two edges can't form a cycle) when considered by Kruskal's Algorithm, so each would be added to the minimum spanning tree.