# CS2223

## Final Exam

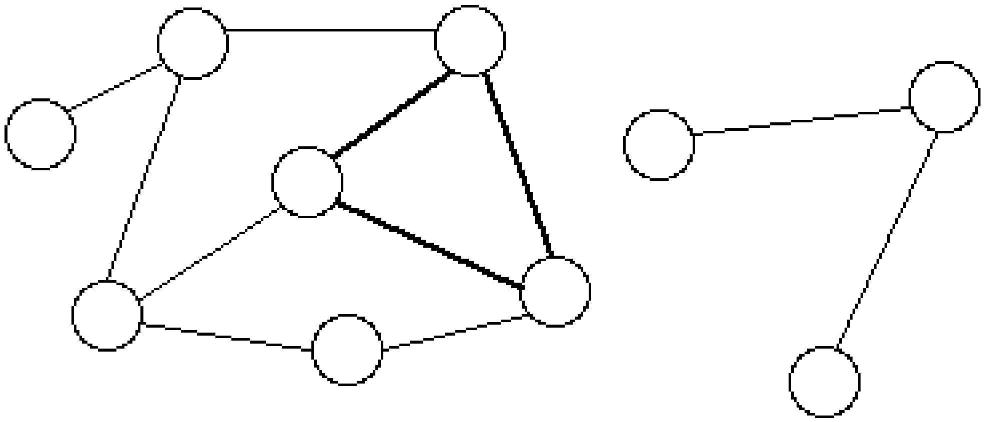
Name__________________

Date: December 19, 1997
All documentation permitted

1. (25 points) Given an array $L[1..n]$ of non-negative reals, we want to find $\max_{1 \le i \le j \le n} \left\{ \prod_{i \le k \le j} L[k] \right\}$, that is, we want to find the maximum product realizable as the product of a contiguous subsequence of numbers of $L$. For example, if $L$=(0.6, 0, 23, 0.5, 28, 0.9), then the answer is 322=23*0.5*28. If $L$=(0.8, 0.9, 0.6), then the answer is 0.9. If
$L$=(1, 0, 12, 3, 1, 2, 0.001, 39), then the answer is 72=12*3*1*2. Describe an $O(n)$ algorithm to solve this problem.

2. (25 points) For undirected graph $G=(N,A)$, we call a vertex $v \in N$ a *centroid* if there is a path from $v$ to every $w \in N$. Describe an algorithm to check if a graph has a centroid. The worst case execution time of your algorithm should be in $O(|N|+|A|) = O(n+a)$.

3. (25 points) The *girth* of an undirected graph is the length of a shortest cycle of the graph. If a graph is acyclic, then its girth is ∞. For example, the graph



has a girth of 3 because of the cycle of length 3 indicated in bold. Describe an algorithm which accepts as input graph $G = (N, A)$ and returns the girth of $G$. Your algorithm should work in time in $O(n^3) = O(|N|^3)$.

4. (25 points) An instance of the THREESET problem provides positive integers $n$ and $B$, and positive integers $x_1,\ldots,x_n$. An algorithm to solve the THREESET problem should return *true* if $x_1,\ldots,x_n$ can be partitioned into multisets $S_1$, $S_2$ and $S_3$ such that $\sum_{x \in S_1} x = \sum_{x \in S_2} x = \sum_{x \in S_3} x = B$ and *false* otherwise. That is, when presented with $n$=8, $B$=9, and integers 1,2,2,2,4,4,5,7, the algorithm should return *true* because of the partition into multisets {4,5}, {2,7} and {1,2,2,4}, and when presented with $n$=8, $B$=9, and integers 1,2,2,2,4,4,5,77, the algorithm should return *false*. Your agorithm should work in time in $O(nB)$ under the standard computational model.

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Solutions to Final Exam

1. *best_to*[1]:=*L*[1]
   **for** *k*:=2 **to** *n* **do**
      *best_to*[*k*]:=max(*L*[*k*], *best_to*[*k*-1]**L*[*k*])

and a (linear time) pass through *best_to* finds and returns its maximum value.

2. A graph has a *centroid* if and only if it is connected. We can use depth-first search (or breadth-first search) which works in time $O(|N|+|A|)$ to test if the graph is connected.

3. Run the Floyd-Warshall algorithm on $G$, such that $L(v,w)=\begin{cases}1, \text{if } \{v,w\}\in A \\ \infty, \text{if } \{v,w\}\notin A\end{cases}$. The girth is the minimum diagonal element of $D$, that is $girth(G)=\min_{1\le k\le n}\{D[k,k]\}$.

4. Given an instance of THREESET, you can call KNAPSACK with capacity $B$ such that the value and weight of object $i$ is $x_i$ for $1\le i\le n$. Using the dynamic programming algorithm to solve KNAPSACK, if there is no packing of value $B$, then THREESET should return *false*. If there is a packing of value $B$, then remove these elements from the set of possible objects. Invoking the knapsack algorithm again, the remaining objects admit a packing of value $B$ if and only if the answer to THREESET is *true*.