1. (25 points) Given an array $L[1..n]$ of non-negative reals, we want to find

$$\max_{1 \leq i \leq j \leq n} \left\{ \prod_{i \leq k \leq j} L[k] \right\},$$

that is, we want to find the maximum product realizable as the product of a contiguous subsequence of numbers of $L$. For example, if $L=(0.6, 0, 23, 0.5, 28, 0.9)$, then the answer is $322=23 \times 0.5 \times 28$. If $L=(0.8, 0.9, 0.6)$, then the answer is $0.9$. If $L=(1, 0, 12, 3, 1, 2, 0.001, 39)$, then the answer is $72=12 \times 3 \times 1 \times 2$. Describe an $O(n)$ algorithm to solve this problem.
2. (25 points) For undirected graph $G=(N,A)$, we call a vertex $v \in N$ a centroid if there is a path from $v$ to every $w \in N$. Describe an algorithm to check if a graph has a centroid. The worst case execution time of your algorithm should be in $O(|N| + |A|) = O(n + a)$. 
3. (25 points) The *girth* of an undirected graph is the length of a shortest cycle of the graph. If a graph is acyclic, then its girth is $\infty$. For example, the graph

![Graph with girth 3](link)

has a girth of 3 because of the cycle of length 3 indicated in bold. Describe an algorithm which accepts as input graph $G = (N, A)$ and returns the girth of $G$. Your algorithm should work in time in $O(n^3) = O(|N|^3)$.
4. (25 points) An instance of the \textsc{ThreeSet} problem provides positive integers \(n\) and \(B\), and positive integers \(x_1, \ldots, x_n\). An algorithm to solve the \textsc{ThreeSet} problem should return \textit{true} if \(x_1, \ldots, x_n\) can be partitioned into multisets \(S_1, S_2\) and \(S_3\) such that
\[
\sum_{x \in S_1} x = \sum_{x \in S_2} x = \sum_{x \in S_3} x = B
\]
and \textit{false} otherwise. That is, when presented with \(n=8, B=9\), and integers \(1,2,2,4,4,5,7\), the algorithm should return \textit{true} because of the partition into multisets \(\{4,5\}, \{2,7\}\) and \(\{1,2,4\}\), and when presented with \(n=8, B=9\), and integers \(1,2,2,2,4,5,7\), the algorithm should return \textit{false}. Your algorithm should work in time in \(O(nB)\) under the standard computational model.
1. best_to[1]:=L[1]
   for k:=2 to n do
       best_to[k]:=max(L[k], best_to[k-1]*L[k])
and a (linear time) pass through best_to finds and returns its maximum value.

2. A graph has a centroid if and only if it is connected. We can use depth-first search (or breadth-first search) which works in time $O(|V|+|A|)$ to test if the graph is connected.

3. Run the Floyd-Warshall algorithm on $G$, such that $L(v,w) = \begin{cases} 1, & \text{if} \{v,w\} \in A \\ \infty, & \text{if} \{v,w\} \notin A \end{cases}$. The girth is the minimum diagonal element of $D$, that is $girth(G) = \min_{1 \leq k \leq n} D[k,k]$.

4. Given an instance of THREESET, you can call KNAPSACK with capacity $B$ such that the value and weight of object $i$ is $x_i$ for $1 \leq i \leq n$. Using the dynamic programming algorithm to solve KNAPSACK, if there is no packing of value $B$, then THREESET should return false. If there is a packing of value $B$, then remove these elements from the set of possible objects. Invoking the knapsack algorithm again, the remaining objects admit a packing of value $B$ if and only if the answer to THREESET is true.