CS2223

FINAL EXAM

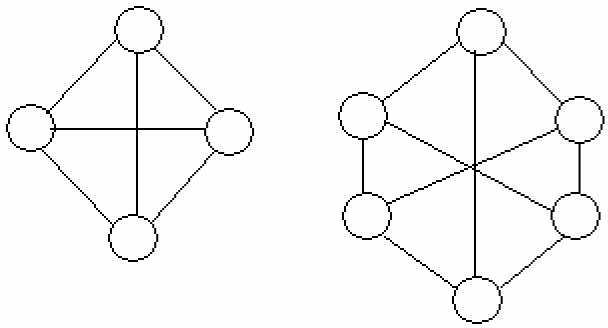
Name_____________________________

DATE: December 18, 2001
All documentation permitted

1. (30 points) Suppose that you are given a weighted graph $G=(N,A)$, vertices $\sigma,\tau\in N$, function $length: A \rightarrow \mathbb{R}^+$, and function $d: N \rightarrow \mathbb{R}^+$ such that for each $v\in N$, $d(v)$ is the length of a shortest path from $\sigma\rightarrow v$. Show how to construct a shortest path from $\sigma\rightarrow\tau$ in time $O(|N|+|A|)$.

2. (35 points) Suppose you are given a directed graph G=(N,A) with function $length$:$A \rightarrow \mathbb{R}^{+}$, and you seek the length of a shortest cycle in G. Find an algorithm with worst-case time complexity in $O(n^3)$ to solve this problem.

3. (35 points) A graph G=(N,A) is a *Maia graph* if it consists of two components and every vertex has degree 3 (every vertex has 3 edges incident with it). For example,



is a Maia graph. Give an algorithm to test if G is a Maia graph. Your algorithm should have worst case execution time in $O(|N|+|A|)$.

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Solutions to Final Exam

1. To print the path in reverse order,

```
v=τ;
while (v!=σ){
    print v;
    for each x adjacent to v     /*  find v's predecessor x on shortest path */
        if (d(v)==d(x)+length(x,v))
            break;               /*  break the for each loop*/
    v=x;
}
print σ;
```

2. Use Floyd's Algorithm, but start the computation with ∞ along the main diagonal, that is, $D[k,k]=\infty$ for $n \geq k \geq 1$. When the algorithm finishes, for each k, $D[k,k]$ is the length of a shortest cycle including node k, and in linear time (time in $O(n)$), we can find the answer, which is the value of $\min_{n \geq k \geq 1} \{D[k,k]\}$

3. Do a depth first search of G to check if it has two components. while each vertex is visited, check that its adjacency list has 3 vertices on it. Return true if and only if all the tests return true.