CS2223

FINAL EXAM

Name______________

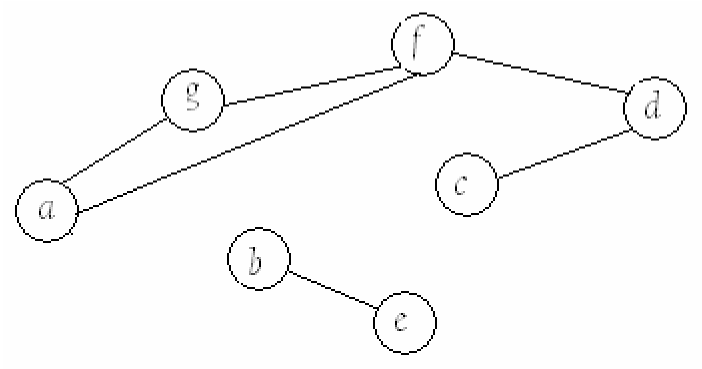
Date: December 14, 2006
All nonelectronic documentation permitted

1	(25)	________
2	(25)	________
3	(25)	________
4	(25)	________

TOTAL ________

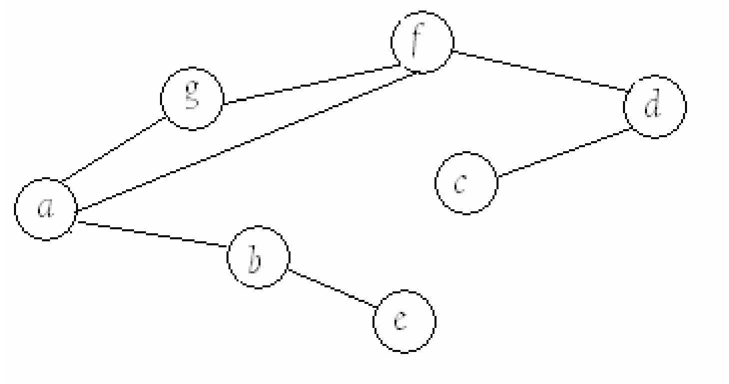
1. (25 points) Design an algorithm that accepts as input graph $G = (V, E)$ and $s, t \in V$ and returns a shortest path from *s* to *t*, where the length of a path is the number of edges on the path. Your algorithm should execute in worst case time in $O(|V| + |E|)$.

For the graph



the shortest path from *g* to *c* is *g*,*f*,*d*,*c*, and there does not exist a shortest path from *g* to *b*.

2. (25 points) The *diameter* of a graph is the distance between two most distant vertices. For example, the diameter of



is 5 because the distance between *c* and *e* is 5, and the distance between all other pairs of vertices is less than 5. Describe an algorithm to compute the diameter of graph $G=(V,E)$ which executes in time in $O(|V|^3)$.

3. (25 points) You manage a supercomputer for m minutes. There are n jobs that want to rent your machine, and they will take $a_1, \ldots, a_n$ minutes respectively. Each job pays you $1 for each minute it uses, but each job must run to completion. How do you schedule the jobs to maximize your profit? Your algorithm should execute in time in $O(mn)$.

4. (25 points) Let $G = (V, E)$ be a weighted graph with $|E| \geq 3$ and $w : E \to \mathbb{Z}^{+}$ with unique edge weights. That is, if $e_0 \neq e_1$, then $w(e_0) \neq w(e_1)$. For each of the following three CLAIMS, tell whether or not the CLAIM is true or false. Justify your reply.

<u>CLAIM 1</u>: The shortest edge of *E* belongs to a minimum spanning tree.

<u>CLAIM 2</u>: The second shortest edge of *E* belongs to a minimum spanning tree.

<u>CLAIM 3</u>: The third shortest edge of *E* belongs to a minimum spanning tree.

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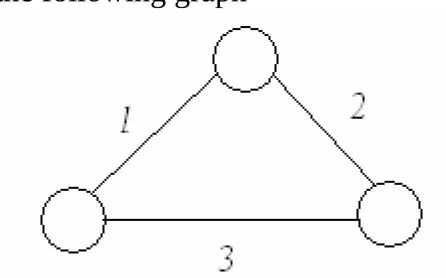
Solutions to <u>FINAL EXAM</u>

1. Do a bfs from *s*. If *t* belongs to the tree, then the path of tree edges from *s* to *t* is a shortest path. If *t* doesn't belong to the tree, then there does not exist a path from *s* to *t*.

2. Run the Floyd-Warshall Algorithm with $w(uv) = \begin{cases} 1, \text{if } uv \in E \\ \infty, \text{otherwise} \end{cases}$, and then return the maximum value in *D*.

3. To decide which jobs to schedule, use the dynamic programming algorithm on an instance of the KNAPSACK PROBLEM for a knapsack with capacity *m* and the i^{th} object having weight **and** value a_i, $1 \le i \le n$.

4. CLAIMs 1 and 2 are true, since Kruskal's algorithm always adds the two shortest edges of *E* to an MST. CLAIM 3 is false for the following graph



the edge of length 3 does not belong to an MST.