CS2223

FINAL EXAM

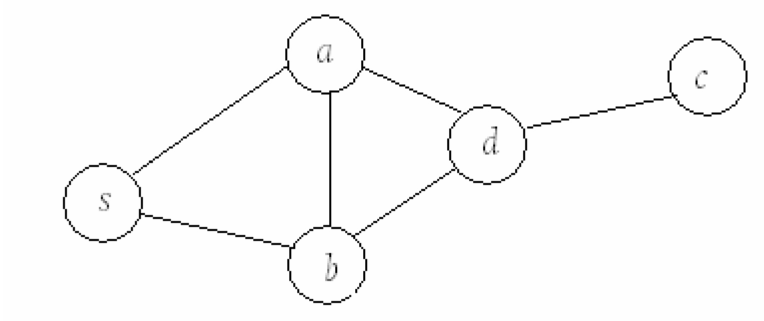
Name_______________

Date: May 2, 2006
All documentation permitted

1	(25)	________
2	(25)	________
3	(25)	________
4	(25)	________

TOTAL _________

1. (25 points) We define the *length* of a path in a graph $G = (V, E)$ to be the number of edges on the path, and the *distance* between two vertices is the length of a shortest path. Describe an algorithm which accepts as input a connected graph $G = (V, E)$ and a source $s \in V$ and which computes the distance between *s* and *v* for every $v \in V$. For graph



the output would be

v	*s*	*a*	*b*	*c*	*d*
distance	0	1	1	3	2

The execution time of your algorithm should be in $O(|V| + |E|)$.

Note: The graph is not a weighted graph. Every edge contributes 1 to the length of a path.

2. (25 points) Let $G = (V, E)$ be a connected weighted graph with weight function $w : E \to \Re^{+}$, and assume that for fixed $v \in V$, the edge $(v, y) \in E$ is the unique edge of minimum weight incident with *v*. That is, for all edges $(v, z) \in E$ with $y \neq z$, $w((v, y)) < w((v, z))$. Either prove that (v, y) must belong to all minimum spanning trees of *G* or provide a counter-example in which there is a minimum spanning tree to which (v, y) doesn't belong.

3. (25 points) Given an array $A[1..n]$ of non-negative reals, we want to find $\max_{1 \le i \le j \le n} \left(\prod_{i \le k \le j} A[k] \right)$. That is, we want to find the maximum product realizable as the product of a contiguous subsequence of numbers of A. For example, if A=(0.6, 0, 23, 0.5, 28, 0.9), then the answer is 322=23*0.5*28. If A=(0.8, 0.9, 0.6), then the answer is 0.9. If A=(1, 0, 12, 3, 1, 2, 0.001, 39), then the answer is 72=12*3*1*2. Describe an algorithm with time complexity in $O(n)$ to solve this problem.

4. (25 points) A graph $G=(V,E)$ has a *bridge* if G is connected and has an edge $e \in E$ whose removal disconnects G. Give an algorithm to test if G has a bridge. The execution time of your algorithm should be in $O\left(|E|*\left(|V|+|E|\right)\right)$.

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Solutions to FINAL EXAM

1. Do a breadth-first search from *s*, and the number of edges on the path in the bfs-tree from *s* to any $v \in V$ is the distance from *s* to *v*.

2. Given the conditions of the problem, (v, y) must belong to all minimum spanning trees of *G*. If not, there must be a minimum spanning tree *T* of *G* which doesn't contain (v, y). Adding (v, y) to *T* yields a connected graph with exactly one cycle, and *v* is on the cycle. The other edge of the cycle incident with *v* has a higher weight than (v, y). So removing that edge from $T \cup \{(v, y)\}$ weighs less than *T* and is a spanning tree, contradicting the fact that *T* is a minimum spanning tree.

3. *best_to*[1] ← *A*[1]
   for *k* ← 2 **to** *n* **do**
      best_to[*k*] ← max(*A*[*k*], *best_to*[*k*-1]***A*[*k*])

4. Do a dfs of *G* to test if it is connected.
      if *G* is not connected **then return** *G* does not have a bridge
   for each $e \in E$ test if *G*-*e* is connected
      if it is not **then return** *G* has a bridge
   return *G* does not have a bridge