CS2223

FINAL EXAM

Name_____________

Date: December 16, 2004
All documentation permitted

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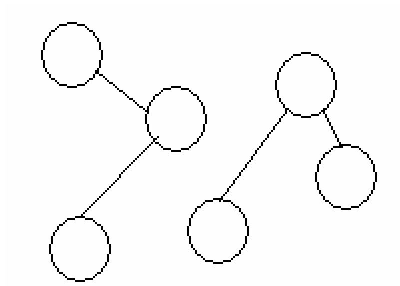
2	________
3	________
4	________
TOTAL	_________

1. (25 points) Describe an algorithm to test if a graph $G = (V, E)$ has a cycle. For example, for the graph

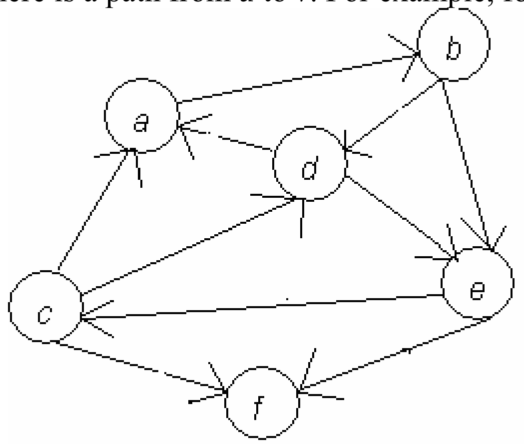


your algorithm should return `true`. The worst case execution time of your algorithm should be bounded by $O(|V| + |E|)$.

2. (25 points) Prove or give a counterexample to the following.

<u>CONJECTURE</u>: For any weighted graph $G=(V,E)$ with $w:E \rightarrow \Re^{+}$ such that the weights of all edges are distinct (that is, $e \neq e' \Rightarrow w(e) \neq w(e')$), and for any pair of vertices $v,w \in V$, all the edges of a shortest path from *u* to *v* must belong to a minimum spanning tree of *G*.

3. (25 points) Suppose that you are given a directed graph $G=(V,E)$, and you want to determine for each pair of vertices $u, v \in V$ whether there is a path from u to v. In particular, we want to compute entries for the Boolean array $P : V \times V \to \{\text{true}, \text{false}\}$ such that for each $u, v \in V$, $P[u,v]$=true if and only if there is a path from u to v. For example, for the graph



$P[a,c]$= true because of the path (a,b), (b,e), (e,c), but $P[f,c]$= false because there does not exist a path from f to c. Describe an algorithm to compute P with worst-case time complexity in $O\left(|V|^3\right)$.

4. (25 points) For the *Knapsack Problem*, we are given positive integer n and n objects with positive weights $w_1,\ldots w_n$ and positive values $v_1,\ldots,v_n$, as well as a knapsack with positive capacity W. We seek a set of integers $x_1,\ldots,x_n$ such that x_i is 0 or 1 for $1\leq i\leq n$ to maximize $\sum_{n\geq i\geq 1} x_i v_i$ subject to $\sum_{n\geq i\geq 1} x_i w_i \leq W$. Consider the following divide-and-conquer strategy to solve the Knapsack Problem (you may assume that W is a power of 2):

- If W=1 and there is at least one object of weight 1, then add a most valuable object of weight 1 to the knapsack and remove it from the set of possible objects.
- If W>1, then solve two Knapsack Problems with knapsacks of size W/2 and combine the two solutions.

Prove or give a counterexample to the following

<u>CONJECTURE</u>: The above algorithm is guaranteed to give an optimal solution to any instance of the Knapsack Problem.

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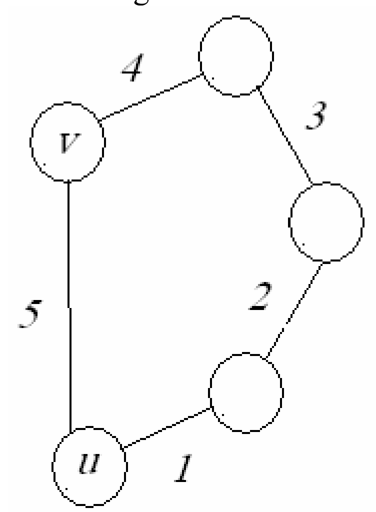
Solutions to Final Exam

1. Do a depth first search of G, which has a cycle if and only if the depth first search forest has a backedge.

2. The CONJECTURE is false. The edge between u and v in the following graph, which **is** the shortest path from u to v, does not belong to the minimum spanning tree.



3. Apply the Floyd-Warshall algorithm to the binary array with 0's along the diagonal ($D[u,u]=0$ for all $u \in V$) and for $u \neq v$, $D[u,v]=1$ if $(u,v) \in E$ and ∞ otherwise). After applying the Floyd-Warshall algorithm, we obtain P from D, by setting $P[u,v]=$true if $D[u,v]$ is finite and false otherwise.

4. The algorithm rarely gives a correct solution. In fact, since it can only add objects of weight 1 it fails for the example $n=1$, $w_1=2$, $v_1=1$, $W=2$ since it will never add the object.