1. (6 points) A typical dynamic programming algorithm provides the cost of a solution or establishes the existence of a solution without actually constructing the solution. To see how to construct a solution by using an efficient mechanism which tests for the existence of a solution, solve the following:
   You are given a boolean function \( \text{BlackBox} \) of two inputs:
   - a list of integers \( x_1, x_2, \ldots, x_n \),
   - an integer \( q \)
and you are told that, in time \( O(1) \), \( \text{BlackBox} \) will return "true" if and only if there is some subset of \( x_1, x_2, \ldots, x_n \) whose sum is \( q \). Design an algorithm (a program is not needed) with the same input which will return an actual subset of \( x_1, x_2, \ldots, x_n \) whose sum is \( q \) if such a subset exists, or it should return "failure".
   For example, \( \text{BlackBox}(23, 27, 41, 72, -4, 6), 29) \) would return "true", but your algorithm with the same input would return \( 27, -4, 6 \). Your algorithm may call \( \text{BlackBox} \) as often as it wishes and it should work in time \( O(n) \).

2. (3 points) For the graph of Figure 9.19 of our text, show how the algorithm identifies articulation points assuming the search starts at node 1 and that nodes of adjacency lists are stored in increasing order. Show the dfs-tree, as well as all values of \( \text{prenum} \) and \( \text{highest} \). Also, identify all articulation points.

3. (5 points) An element \( x \) is a majority element of array \( A[n] \) if at least half the elements in \( A \) are \( x \). Thus, \( A=(0, -1, 5, 5, 5, 6, 5) \) has majority element 5, and \( A=(5, -1, 0, 12, 5, 6, 5) \) does not have a majority element. Design a probabilistic algorithm (precise pseudo-code suffices) to test if \( A \) has a majority element. Your algorithm should work in time \( O(n) \). If your algorithm says that \( A \) has a majority element, it should be correct, and if your algorithm says that \( A \) does not have a majority element, then for any array \( A \) it may be wrong with probability \( \frac{1}{2} \).