CS2223
HW#7

DUE: Tuesday, December 12

1. (10 points) For $n \geq 1$, the *n*-queens problem asks for *Q*(*n*), the number of ways to place *n* nonattacking queens on an $n \times n$ chessboard, where two queens attack each other if they are on the same row, column or diagonal. The URL

http://www.research.att.com/cgi-bin/access.cgi/as/njas/sequences/eisA.cgi?Anum=000170

shows the first few values of *Q*(*n*) to be:

n	*Q*(*n*)
1	1
2	0
3	0
4	2
5	10
6	4
7	40
8	92
9	352
10	724
11	2680
12	14200
13	73712
14	365596
15	2279184
16	14772512
17	95815104
18	666090624
19	4968057848
20	39029188884
21	314666222712
22	2691008701644
23	24233937684440

Write and test a probabilistic program to estimate *Q*(*n*) for arbitrary *n*. Show and discuss the results of your tests.

2. (7 points) A herpetologist has collected *n* salamanders, which are believed to belong to two different species. She examines *m* pairs, $m \leq \binom{n}{2}$, and labels the examined pairs as either the *same* or *different*, depending upon whether she thinks they're the same species or different. We want to test whether her decisions are consistent. For example, if she judged that:

- *a* and *b* are the *same*,
- *a* and *c* are the *same*,
- *b* and *c* are *different*,

then her decisions are inconsistent. Show how to test, in $O(n+m)$ time, whether or not her decisions are consistent.

CS2223
HW#7 SOLUTIONS

2. We consider the graph with the *n* salamanders as vertices, and *m* edges corresponding to the compared pairs of salamanders. We want to know whether the salamanders can be partitioned into sets so that all edges within each set are labeled *same*. We don't care about vertices with no edges incident upon them. Labels can be consistent even if there are more than two clusters internally labeled *same*.

for each $v \in V$
    Mark each v *unvisited*
for each $v \in V$
    if v *unvisited*
        do a depth-first search from v, following all edges
        labeled *same*. If a backedge labeled *different* is encountered,
        then **return "**labeling is inconsistent"
return "labeling is consistent"