CS2223
HW#7

DUE: Monday, December 17

(8 points) An *n-bit code* ψ is a subset of $\{0,1\}^n$. That is, it is a set ψ of *n*-bit sequences called *codewords*. A code ψ is *single error detecting* if every pair of codewords $x, y \in \psi$ differ in at least two positions. An *n*-bit single error detecting code ψ is *maximal* if for every *n*-bit sequence *x*,

$$x \notin \psi \Rightarrow \psi \cup \{x\} \text{ is not single error detecting .}$$

That is, ψ is maximal if no new codeword can be added to ψ without violating single error detectability. For example, $\{0001, 0011, 1011\}$ is a 4-bit code, but it is not single error detecting because 0001 and 0011 differ in only one position. Also, $\psi = \{0000, 0011, 1100, 1111\}$ is a single error detecting 4-bit code, but it is not maximal because $\psi \cup \{0101\}$ is single error detecting.

Codes are used to transmit information over noisy channels, and single error detecting codes are desirable because an error in the transmission of any one bit can be detected (it takes at least two errors to transform a codeword into another codeword), and maximal codes are desirable because this increases the rate at which information can be transmitted.

- Write a program to estimate the number of maximal *n*-bit single error detecting codes.

- For $0 \le k \le 2^{10}$, estimate the number of maximal single error detecting 10-bit codes. It is very difficult to get an exact answer to this question.