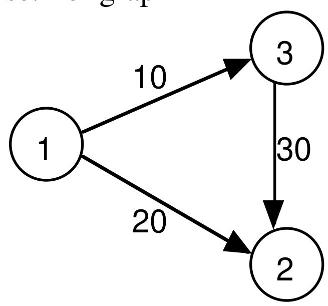
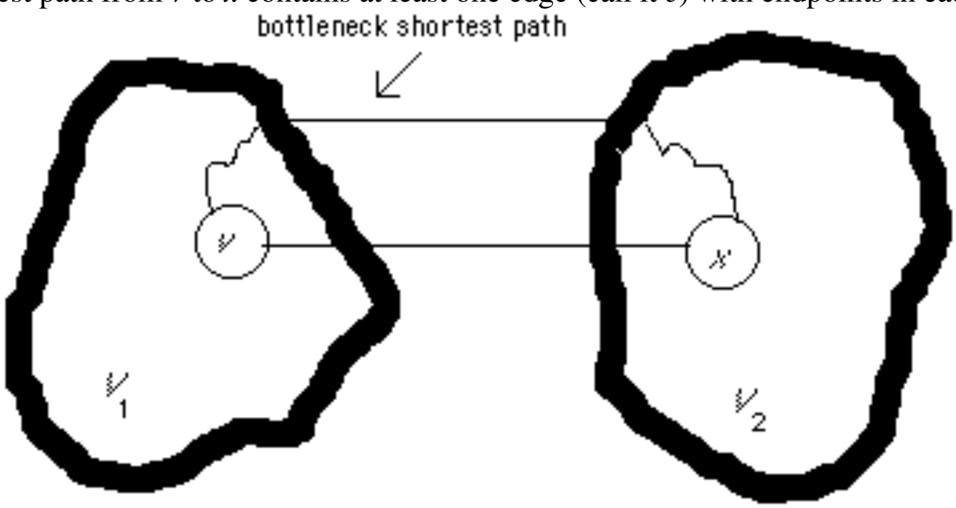
CS2223
HW#5 SOLUTIONS

1. The conjecture is false. For graph



*Dijkstra** would return $D[2]$ to be 20, whereas there is a path of length 40 from 1 to 2.

2. A) Compute and return a minimum spanning tree of G.
B) Suppose that for some pair of vertices v, x in G, the edge $\{v, x\}$ belongs to a minimum spanning tree but is not part of a bottleneck shortest path from v to x. For any partition of the vertices of G into two sets, one of which contains v and one of which contains x, the bottleneck shortest path from v to x contains at least one edge (call it e) with endpoints in each set.



But by the definition of the MST, $L(\{v,x\}) \leq L(e)$ (or else a cheaper tree could be obtained by replacing e by edge $\{v, x\}$). So edge e could be replaced be edge $\{v, x\}$ in the tree, without increasing the length of any bottleneck shortest path. Continuing this way, the MST must be an acceptable answer.