

CS2223
HW#5

DUE: Friday, April 14

1. (12 points) For each of the following CONJECTURES, either state that it is true or provide a counterexample showing that it is false.

CONJECTURE A: If every edge of connected weighted graph $G = (V, E)$ has a distinct weight (that is, $e, e' \in E \wedge e \neq e' \Rightarrow w(e) \neq w(e')$), then G admits exactly one minimum spanning tree.

CONJECTURE B: If e is a minimum weight edge of connected weighted graph $G = (V, E)$, then e must belong to every minimum spanning tree of G .

CONJECTURE C: If $G = (V, E)$ is a connected weighted graph and there exist $e, e' \in E$ with $w(e) = w(e')$, then G must admit at least two distinct minimum spanning trees.

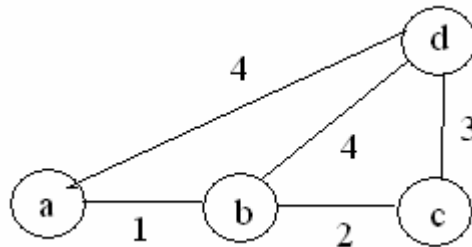
2. (5 points) Do **Exercise 23.2-8** on pg. 574 of our text.

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HW#5 SOLUTIONS

1. CONJECTURE A is true. As shown in class, each minimum weight edge added to the forest must belong to an MST, and, by induction, each of the $n-1$ edges must belong and the MST is unique.

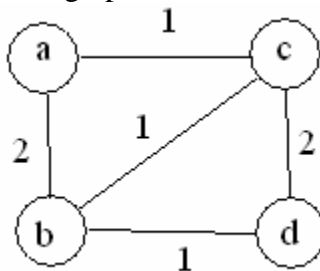
CONJECTURE B is true for the same reason.

CONJECTURE C is false. Consider the tree



The edges of lengths 1, 2 and 3 belong to the unique MST.

2. Professor Toole is wrong. For the graph



and partition $V_1 = \{a, b\}$, $V_2 = \{c, d\}$, the tree constructed by Toole's algorithm contains both edges of weight 2 as well as one edge of weight 1, whereas the MST consists of the three edges of weight 1.