CS2223
HW#5

DUE: Monday, December 6

1. (10 points) Suppose you are given an infinite supply of coins of denominations $\{c_1, \ldots, c_n\}$.

For example, in the US (ignoring silver dollars) n=5 and $c_1 = 1, c_2 = 5, c_3 = 10, c_4 = 25$ and c_5=50. Design an algorithm to compute the number of ways to make change for a¢. For example, for input

$$n = 5, \{1, 5, 10, 25, 50\}, \ a=17$$

the output would be 6 because of the solutions

$$(10,5,1,1), (10,1,1,1,1,1,1,1), (5,5,5,1,1), (5,5,1,1,1,1,1,1,1),$$
$$(5,1,1,1,1,1,1,1,1,1,1,1,1), (1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1)$$

2. (15 points) Do **Problem *16-1***, parts ***a***, ***c*** and ***d*** on pg. 402 of our text.

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HW#5 SOLUTIONS

1. For $0 \le k \le n,\ 0 \le x \le a$, we let $f(k,x)$ denote the number of ways to make change for x¢ using coins of denominations $\{c_1,...,c_k\}$. We assume that $f(k,x)=0$ if $k<0$ or $x<0$. We note that we can use 0,1,...,$\lfloor x/c_k \rfloor$ coins of denomination c_k. If we use exactly i coins of denomination c_k, then there are $f(k-1, x-ic_k)$ ways to make change for ($x-ic_k$)¢ using coins of denominations $\{c_1,...,c_{k-1}\}$.

$$f(k,x) = \sum_{0 \le i \le \lfloor x/c_k \rfloor} f(k-1, x-ic_k)$$

for $x \leftarrow 1$ **to** a **do**
  for $k \leftarrow 0$ **to** n **do**
    $f(k,x) \leftarrow f(k-1,x)$ **/* using 0 c_k */**
    for $i \leftarrow 1$ **to** $\lfloor x/c_k \rfloor$ **do**
      $f(k,x) \leftarrow f(k,x) + f(k-1, x-ic_k)$ **/* using i c_k */**
return $f(n,a)$

2. ***a***
- Give $q=\lfloor n/25 \rfloor$ quarters
- Give $d = \lfloor (n-q*25)/10 \rfloor$ dimes
- Give $r = \lfloor (n-q*25-d*10)/5 \rfloor$ nickels
- Give $n-q*25-d*10-r*5$ pennies

c Let n=6¢, and let the denominations be 1¢, 3¢ and 4¢. The Greedy Algorithm of part ***a*** uses three coins, 4¢, 1¢ and 1¢, although the optimal answer uses the two coins 3¢ and 3¢.

d For $0 \le k \le n,\ 0 \le x \le a$, we let $f(k,x)$ denote the smallest number of coins of denominations $\{c_1,...,c_k\}$ to make change for x¢. We assume that $f(k,x)=0$ if $k<0$ or $x<0$. We note that we can use 0,1,...,$\lfloor x/c_k \rfloor$ coins of denomination c_k. If we use exactly i coins of denomination c_k, then the fewest coins to make change for ($x-ic_k$)¢ using coins of denominations $\{c_1,...,c_{k-1}\}$, is $f(k-1, x-ic_k)$.

$$f(k,x) = \min_{0 \le i \le \lfloor x/c_k \rfloor} \left(i + f(k-1, x-ic_k)\right)$$

for $x \leftarrow 1$ **to** a **do**

  for $k \leftarrow 0$ **to** n **do**

    $f(k,x) \leftarrow f(k-1,x)$ **/* using 0 c_k */**

    for $i \leftarrow 1$ **to** $\lfloor x / c_k \rfloor$ **do**

      $f(k,x) \leftarrow \min\left(f(k,x), i + f(k-1, x - ic_k) \right)$ **/* using i c_k */**

return $f(n,a)$