CS2223
HW#5

DUE: Monday, December 8

1. (6 points) Suppose you're living in a country with a monetary system in which the coins have values $\{c_1,\ldots,c_n\}$, where $c_i \in \mathbb{Z}^+$, $1 \le i \le n$. Given $\{c_1,\ldots,c_n\}$ and $m \in \mathbb{Z}^+$, we want to know if it is possible to express m exactly by drawing coins from an infinite supply of coins of each denomination. Fo example, if $\{c_1,\ldots,c_n\} = \{5,7,12\}$, then it is not possible to express m=16 exactly with these coins, but it is possible to express m=22 exactly with these coins, either as 5, 5, 5, 7 or as 5, 5, 12. Show how to solve this problem for any $\{c_1,\ldots,c_n\}$ and m by reducing it to an instance of the KNAPSACK PROBLEM.

2. (4 points) Do **Exercise *25.2-6*** from pg. 635 of our text.

3. (8 points) A recursive definition of the binomial coefficients is that

$$\binom{n}{k} = \begin{cases} 1, \text{ if } k = 0 \vee k = n \\ \binom{n-1}{k-1} + \binom{n-1}{k}, \text{otherwise} \end{cases}$$

The corresponding recursive program is

$C(n,k)$
  if $k = 0 \vee k = n$
    then return 1
    else return $C(n-1,k-1) + C(n-1,k)$

a What is the time to execute $C(n,k)$, as a function of n and k?
b Write a dynamic programming algorithm to compute $C(n,k)$ with an execution time in $\Theta(nk)$.

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HW#5 SOLUTIONS

1. For each coin c_i, we make $\lfloor W / c_i \rfloor$ copies of the coin. Choosing W=m and $v_{i,j} = w_{i,j} = c_i, 1 \le i \le n, 1 \le j \le \lfloor W / c_i \rfloor$, we note that it is possible to express m exactly with the coins if and only if there is a packing of the knapsack of value m=W.

2. The graph admits a negative length cycle if and only if there is an $i,\ 1 \le i \le n$, such that $d_{ii}^{(n)} < 0$.

$negativecycle \leftarrow$ false
for $i \leftarrow 1$ **to** n **do**
  if $d_{ii}^{(n)} < 0$ **then** $negativecycle \leftarrow$ true
return *negativecycle*

3. ***a*** Because ultimately the recursion tree causes $\binom{n}{k}$ 1's to be added together, computing $C(n,k)$ makes $\binom{n}{k}$ calls on C, and has a time complexity $\Theta\left(\binom{n}{k}\right)$.

b The values are stored in an array $C[0..n, 0..k]$.

for $i \leftarrow 0$ **to** n **do**
  for $j \leftarrow 0$ **to** $\min(i,k)$ **do**
    if $k = 0 \vee k = n$
    then $C[i, j] \leftarrow 1$
    else $C[i, j] \leftarrow C[i-1, j-1] + C[i-1, j]$