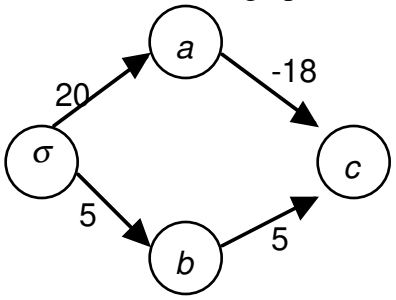
CS2223
HW#4 SOLUTIONS

1. The conjecture is true. If all edge lengths are positive, there's nothing to prove. Let e_0 be a least cost edge of A. Add $|L(e_0)|$ to each length of A, that is, define $L':A \to R^+$ by $L'(e) = L(e) + |L(e_0)|$. We know that the algorithm works correctly for lengths L', since all lengths are non-negative. But since L' imposes the exact same order on A as L, the algorithm will follow exactly the same sequence of steps for L as for L', and would construct exactly the same spanning tree, which must have a minimal length.

2. The conjecture is false. Consider the graph



The shortest path from σ to c is σ,a,c of length 2, although Dijkstra's algorithm would construct the path σ,b,c of length 10.

3. (*a*) The greedy algorithm is optimal. Assume without loss of generality that $s_i \le s_{i+1}$, for $1 \le i < n$. There is a $j \le n$ such that the greedy algorithm packs programs $P_1,\ldots,P_j$. If this packing were not optimal, there must be a larger packing P such that there exist k, l with $1 \le k \le j$ and $P_k \notin$ P and $j < l \le n$ and $P_l \in$ P. But $s_k \le s_l$,, so P can be modified by replacing P_l by P_k and the modified takes no more space. Continuing in this matter, all of $P_1,\ldots,P_j$ can be added to P, but since nothing else fits along wih $P_1,\ldots,P_j$, we contradict the assumption that a better packing fits. Hence the greedy algorithm must be optimal.
(*b*) The greedy algorithm is not optimal. If the disk has capacity D=9 and the n=3 programs have sizes 4, 5 and 6, the greedy algorithm will only fit the program of size s_3=6 for a utilization of 2/3, whereas an optimal packing (of the smaller two programs) yields a utilization of 1.