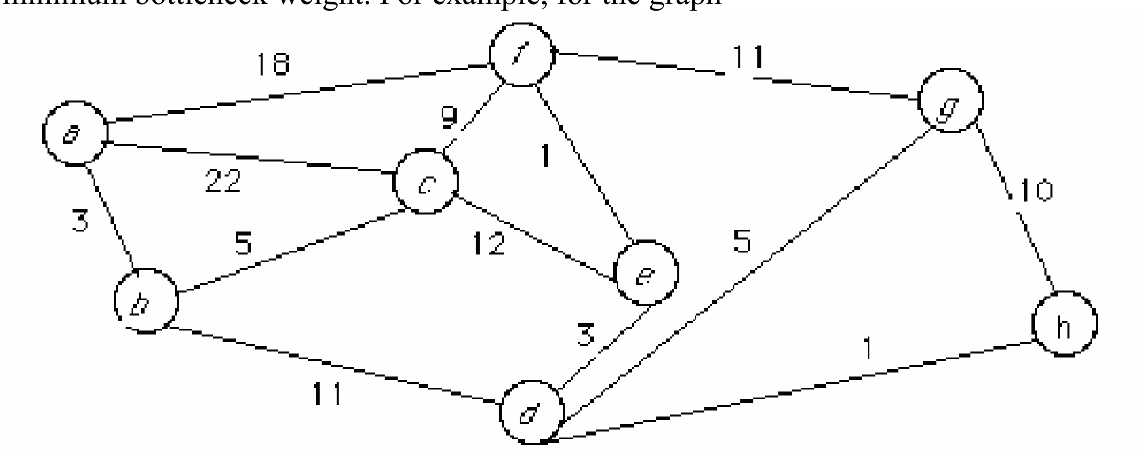
CS2223
HW#4

DUE: Monday, November 29

1. (5 points) Do **Exercise *23.1-10*** on page 567 of our text.

2. (12 points) For a graph *G* with weight function $w: E \rightarrow \Re^{+}$ and for any pair of vertices $v, x \in V$, we define the *bottleneck weight* of a path *P* from *v* to *x* as the weight of the longest edge on the path *P*, and a *bottleneck shortest path* from *v* to *x* as a path from *v* to *x* of minimum bottleneck weight. For example, for the graph



the path (*a*, *b*, *c*, *f*, *e*, *d*, *g*) from *a* to *g* has bottleneck weight 9, the path (*a*, *f*, *e*, *d*, *g*) from *a* to *g* has bottleneck weight 18, and the path (*a*, *b* ,*d*, *h*) from *a* to *h* has bottleneck weight 11. The *bottleneck shortest path* from *a* to *h* is (*a*, *b*, *c*, *f*, *e*, *d*, *h*).

a Describe an algorithm (precise pseudo-code will suffice) which will accept as input a weighted connected graph *G*, and will construct a connected subgraph of *G* which contains a unique *bottleneck shortest path* from any vertex of *G* to any other vertex of *G*. Your algorithm's execution time should be bounded by $O(|E|^2)$.

b Show that your algorithm works correctly.

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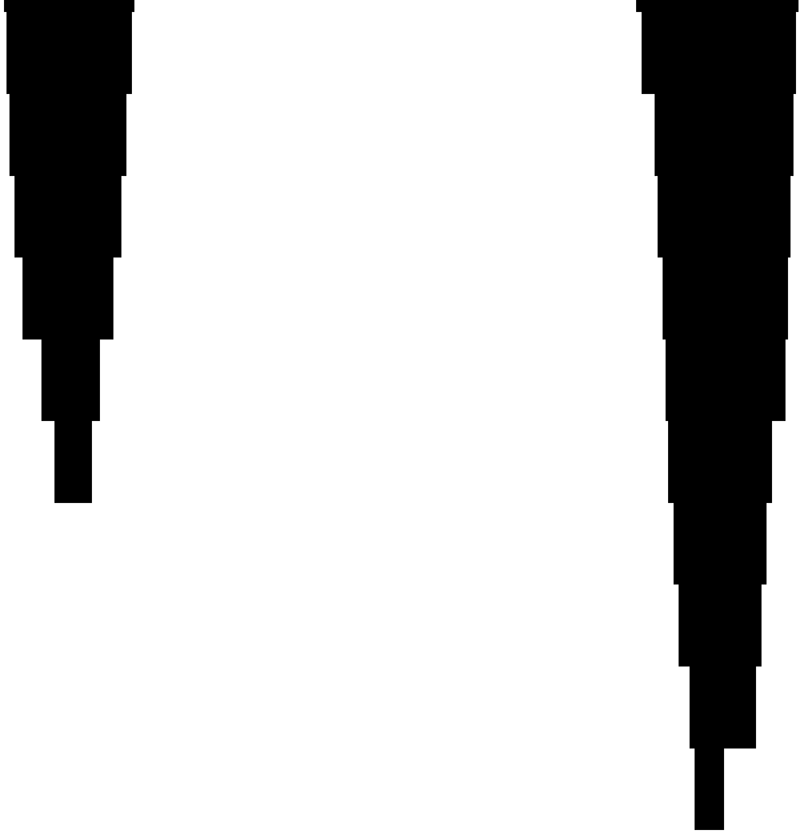
HW#4 SOLUTIONS

1. We consider running Kruskal's algorithm on the modified graph with weights *w*'. The relative order between the edges of *T* is maintained, so Kruskal's algorithm treats them in the same order. For any edge $e \in E - T$, the forest T^* constructed by Kruskal contains all the edges it contained with weight *w*, and possibly more. Thus $T^* \cup \{e\}$ contains a cycle with weight *w*', and *e* will still not be added to T^*.

2. ***a*** Compute and return a minimum spanning tree of *G*.

b Suppose that for some pair of vertices *v*, *x* in *G*, the edge (*v*, *x*) belongs to a minimum spanning tree but is not part of a bottleneck shortest path from *v* to *x*. For any partition of the vertices of *G* into two sets, one of which contains *v* and one of which contains *x*, the bottleneck shortest path from *v* to *x* contains at least one edge (call it *e*) with endpoints in each set.



But by the definition of the MST, $w(\{v,x\}) \le w(e)$ (or else a cheaper tree could be obtained by replacing e by edge $\{v, x\}$). So edge e could be replaced be edge $\{v, x\}$ in the tree, without increasing the cost of any bottleneck shortest path. Continuing this way, the MST must be an acceptable answer.