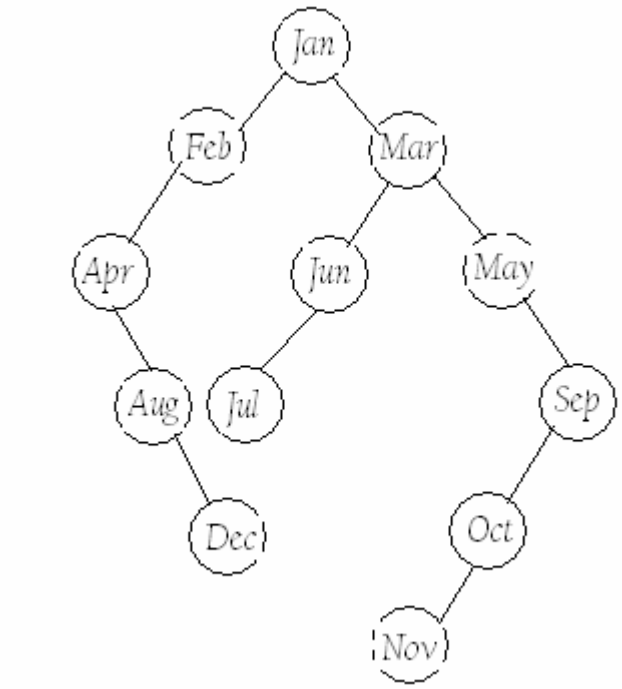
CS2223
HW#3

DUE: Friday, November 10

A *binary search tree*, covered in Chapter 12 of our text, can be used as a data structure to realize a (max)-priority queue. The binary search tree after the sequence of INSERTs

$$\langle Jan,\ Feb,\ Mar,\ Apr,\ May,\ Jun,\ Jul,\ Aug,\ Sep,\ Oct,\ Nov,\ Dec \rangle$$

is



For the following problems, we assume a binary search tree implementation of a (max)-priority queue Q.

1. (4 points) If Q contains n elements, what is a worst-case time (using Θ-notation) to implement an EXTRACT-MAX(Q)? Describe an input sequence of keys such that

$$\text{INSERT}(Q,x_1),\ \text{INSERT}(Q,x_2),\ldots,\ \text{INSERT}(Q,x_n)$$

yields a worst-case tree.

2. (4 points) For your sequence from part 1 above, how many nodes are probed (touched) in executing the sequence of n INSERTs? For example, starting with an empty (max)-priority queue Q and then executing

INSERT(Q,275), INSERT(Q,659)

0 nodes would be probed in INSERTing 275 and 1 node would be probed in INSERTing 659, for a total of 1 node probed. Give an exact answer, not using asymptotic notation. For n=2 your answer should be 1.

3. (14 points) Describe programs to implement INSERT(Q,x), MAXIMUM(Q) and EXTRACT-MAX(Q). You should like to estimate the average-case time, as a function of n, to execute INSERT(Q,x) and EXTRACT-MAX(Q). We assume that the elements in the tree are all distinct and that each permutation of the inputs is equally likely.

- For each of several values of n
  - Grow a tree by INSERTing n random numbers
  - For 1000 random keys y determine the times to execute the pair
    INSERT(Q,y)
    EXTRACT-MAX(Q)

Determine the dependence on n of the execution times to INSERT and EXTRACT-MAX.

CS2223
HW#3 SOLUTIONS

1. $\Theta(n)$ time is required after input sequence INSERT(Q,1), INSERT(Q,2),..., INSERT(Q,n).

2. $$\sum_{1\le k\le n}(k-1)=\sum_{1\le k\le n}k-\sum_{1\le k\le n}1=\frac{n(n+1)}{2}-n=\frac{n(n-1)}{2}$$