CS2223
HW#2 Solutions

1. Guess (and verify) in linear time that A is sorted. If A is sorted, return the middle value, otherwise sort it and return the middle value.

> *sorted* $\leftarrow$ true
> **for** $k \leftarrow 1$ **to** n-1 **do if** $A[k]>A[k+1]$**then** *sorted* $\leftarrow$ false
> **if not** *sorted* **then** INSERTIONSORT(A)
> **return** $A[(n+1)/2]$

2. $\{(1/3)^n\}$,{lg lg$n$}, { lg$n$, ln$n$},{ $n^{1/2}$, $\sqrt{n}+\lg n$ },{ $\sqrt{n}*(\lg n)^2$ },{n},
{ n*lg$n$ },{ n^2, $n^2+\lg n$ },{ n^{34} },{ 2^n },{$n!$}

3. (a) The Conjecture is true. For any f, choose $n_0=1$ and c=1. For all $n \geq 1$ it follows that $f(n) \leq f(n)$.

(b) The Conjecture is false. Choosing $f(n)=1$ and $g(n)=n$, we see that $c_0=1$ and $n_0=1$ yields that for all n=1, 1=1*n. Hence $f(n) \in O(g(n))$. But in the other direction, if $g(n) \in O(f(n))$, then there must exist c_1 and n_1 such that for all $n=n_1$, $g(n)=n=c_1*f(n)=c_1$. Choosing $n=\max\{c_1, n_1\}+1$ forces a contradiction.

(c) Since $f(n) \in O(g(n))$) and $g(n) \in O(h(n))$, then there exist c_0, n_0, c_1, n_1, such that $f(n)=c_0*g(n)$ and $g(n)=c_1*h(n)$ for all $n=\max\{n_0,n_1\}$. But $f(n)=c_0*g(n)=c_0*c_1*h(n)$ for all $n=\max\{n_0,n_1\}$. Letting $n_2=\max\{n_0,n_1\}$ and $c_2=c_0*c_1$ assures that $f(n) \in O(h(n))$.