CS2223
HW#2 Solutions

1. Since the values of the $2n$ numbers are restricted to fall in the range $\{1,...,10n\}$, they can be sorted in worst-case linear time by the procedure *pigeonhole* on page 69 of our text. Then, again in linear time, the largest n numbers can be placed in *Haves* and the smallest n in *HaveNots*.

2 ***a*** $n^4 + 1000000n^3 \in \Theta(n^4)$ is true. To show that $n^4 + 1000000n^3 \in O(n^4)$, we choose $n_0 = 0$ and c=1000001. For all $n \geq 0$, $n^4 + 1000000n^3 \leq n^4 + 1000000n^4 = 1000001n^4$. To show that $n^4 \in O(n^4 + 1000000n^3)$, choose $n_0 = 0$ and c=1. For all $n \geq 0$, $n^4 \leq n^4 + 1000000n^3$.
b $(n+1)! \in \Theta(n!)$ is false. If it were true, then we'd need $(n+1)! \in O(n!)$, which would imply the existence of n_0 and c such that for all $n \geq n_0$, $(n+1)! \leq cn!$. Dividing both sides by $n!$ would yield that for all $n \geq n_0$, $n+1 \leq c$, which is impossible.
c $n^2 \in O(n^2 \lg n)$ is true. Choose $n_0 = 1$ and c=1. For all $n \geq 0$, $1 \leq \lg n$ which implies that $n^2 \leq n^2 \lg n$.
d $n^2 \in \Theta(n^2 \lg n)$ is false. If it were true, then we'd need $n^2 \lg n \in O(n^2)$, which would imply the existence of n_0 and c such that for all $n \geq n_0$, $n^2 \lg n \leq cn^2$. Dividing both sides by n^2 would yield that for all $n \geq n_0$, $\lg n \leq c$, which is impossible since the function $\lg n$ goes to ∞ as n goes to ∞.

3. You can't change the **constant** each time through the induction step. The induction hypothesis claimed the existence of a **constant** c, and then we changed it to a new constant $\hat{c} = c + 2$.

4. $O(n \log n) \subset O(n^{1+\varepsilon}) \subset O(n^2 / \log n) \subset O(n^8) = O((n^2 - n + 1)^4) \subset O((1+\varepsilon)^n)$