1. Since the values of the $2n$ numbers are restricted to fall in the range $\{1, \ldots, 10\}$, they can be sorted in worst-case linear time by the procedure pigeonhole on page 69 of our text. Then, again in linear time, the largest $n$ numbers can be placed in $Haves$ and the smallest $n$ in $HaveNots$.

2. a. $n^4 + 1000000n^3 \in O(n^4)$ is true. To show that $n^4 + 1000000n^3 \in O(n^4)$, we choose $n_0 = 0$ and $c=1000001$. For all $n \geq 0$, $n^4 + 1000000n^3 \leq n^4 + 1000000n^4 = 1000001n^4$. To show that $n^4 \in O(n^4 + 1000000n^3)$, choose $n_0 = 0$ and $c=1$. For all $n \geq 0$, $n^4 \leq n^4 + 1000000n^3$.

b. $(n+1)! \in \Theta(n!)$ is false. If it were true, then we’d need $(n+1)! \in O(n!)$, which would imply the existence of $n_0$ and $c$ such that for all $n \geq n_0$, $(n+1)! \leq cn!$. Dividing both sides by $n!$ would yield that for all $n \geq n_0$, $n+1 \leq c$, which is impossible.

c. $n^2 \in O(n^2 \log n)$ is true. Choose $n_0 = 1$ and $c=1$. For all $n \geq 0$, $1 \leq \log n$ which implies that $n^2 \leq n^2 \log n$.

d. $n^2 \in \Theta(n^2 \log n)$ is false. If it were true, then we’d need $n^2 \log n \in O(n^2)$, which would imply the existence of $n_0$ and $c$ such that for all $n \geq n_0$, $n^2 \log n \leq cn^2$. Dividing both sides by $n^2$ would yield that for all $n \geq n_0$, $\log n \leq c$, which is impossible since the function $\log n$ goes to $\infty$ as $n$ goes to $\infty$.

3. You can’t change the constant each time through the induction step. The induction hypothesis claimed the existence of a constant $c$, and then we changed it to a new constant $\hat{c} = c + 2$.

4. $O(n \log n) \subset O(n^{1+\varepsilon}) \subset O(n^2 / \log n) \subset O(n^8) = O\left((n^2 - n + 1)^4\right) \subset O\left((1 + \varepsilon)^n\right)$