1 (8 points) Assume you are given an array \( A \) of \( n \) integers, and the desired output is the median of \( A \). You may assume that \( n \) is odd. (The median of \( A \) is the element of \( A \) such that \( (n+1)/2 \) elements of \( A \) are greater than or equal to the median and \( (n+1)/2 \) elements of \( A \) are less than or equal to the median). For example, if \( n=5 \) and \( A = \{-25, 14, 22, -10, -558\} \), then the median is \(-10\). Assuming the benchmark computational operation is a pairwise comparison, describe an algorithm (you don’t need to write a program) to solve this problem which in the best-case uses \( n-1 \) pairwise comparisons.

2. (8 points) Group the following 14 functions so that \( f \) and \( g \) are in the same group if and only if \( f \in \Theta(g) \). List the groups from lowest order to highest order.

\[
\begin{array}{cccc}
n & n^{34} & 2^n & n \cdot \log n \\
\log n & \sqrt{n + \log n} & \log \log n & \sqrt{n \cdot (\log n)^2} \\
(1/3)^n & n! & n^2 & n^2 + \log n \\
\ln n & n^{1/2} & & \\
\end{array}
\]

3. (6 points) The statement \( f(n) \in O(g(n)) \) can be viewed as a relation between functions \( f \) and \( g \). Prove or give a counterexample to each of the following conjectures that this relation is reflexive, symmetric and transitive.

(a) **Conjecture 1**: For any (univariate) function \( f, f(n) \in O(f(n)) \).
(b) **Conjecture 2**: For any (univariate) functions \( f, g \), if \( f(n) \in O(g(n)) \), then \( g(n) \in O(f(n)) \).
(c) **Conjecture 3**: For any (univariate) functions \( f, g, h \), if \( f(n) \in O(g(n)) \) and \( g(n) \in O(h(n)) \), then \( f(n) \in O(h(n)) \).