# CS2223
# HW#1

**DUE**: Tuesday, October 31

1. Consider the set $\{0,1\}^n$ for $n \geq 0$, where for *n*=2 we have

$$\{0,1\}^2 = \{00, 01, 10, 11\}.$$

***a*** (1 point) What is the cardinality of $\{0,1\}^n$?

***b*** (5 points) Prove or give a counterexample to the following. Justify your response.

<u>CONJECTURE</u>: For any $n \geq 0$ the elements of $\{0,1\}^n$ can be listed in a sequence such that each adjacent pair of elements in the sequence differ in exactly **one** position, and for $n > 0$ the first and last elements of the sequence differ in exactly **one** position?

For example, an acceptable sequence for *n*=2 would be $(00, 01, 11, 10)$. The sequence $(00, 01, 10, 11)$ would not be an acceptable sequence since 01 and 10 differ in two positions, and also 00 and 11 differ in two positions. Hint: Think recursively in *n*.

2 (10 points) There is one sequential processor and a set of *n* processes, $\{p_1, ..., p_n\}$, with dependencies on the execution of the processes. For example, dependency $(p_i, p_j)$ means that $p_i$ must execute before $p_j$. A *deadlock* is a sequence of processes $(p_{i_1}, ..., p_{i_k})$ such that $(p_{i_l}, p_{i_l+1})$ is a dependency for $1 \leq i_l < k$ and $(p_{i_k}, p_{i_1})$ is a dependency. A *schedule* is a permutation of the processes $(p_{j_1}, ..., p_{j_n})$ such that for all $1 \leq j_l \leq j_k \leq n$ there does not exist a dependency $(p_{j_l}, p_{j_k})$. So, for *n*=6 and dependencies $(p_2, p_5), (p_1, p_4), (p_2, p_6), (p_5, p_1), (p_4, p_2)$ the sequence $(p_2, p_5, p_1, p_4, p_2)$ is a deadlock, and for dependencies $(p_2, p_5), (p_1, p_4), (p_2, p_6), (p_4, p_2)$ the sequence $(p_1, p_4, p_2, p_6, p_3, p_5)$ is a schedule. Give pseudocode for a polynomial time algorithm to take as input a value for *n* and a set of dependencies and to determine if either there is a deadlock or a schedule.

# CS2223
## HW#1 SOLUTIONS

1. ***a*** $2^n$

***b*** The CONJECTURE is true. Such a sequence is called a Gray Code. We prove existence of a Gray Code by induction on *n*. As a basis, clearly the sequence (0) is a Gray Code for *n*=0. Assume that $\left(s_1,...,s_{2^n}\right)$ is a Gray Code for fixed *n*. Then $\left(0s_1,...,0s_{2^n},1s_{2^n},...,1s_1\right)$ is a Gray Code for *n*+1. To see this we note that consecutive $(n+1)$-vectors in the subsequence $\left(0s_1,...,0s_{2^n}\right)$ differ from each other in the same bit in which the corresponding differ in $\left(s_1,...,s_{2^n}\right)$. The same holds for the subsequence $\left(1s_{2^n},...,1s_1\right)$. Finally, the pair of vectors $\left(0s_{2^n},1s_{2^n}\right)$ differ only in the leftmost bit, as well as the pair of vectors $\left(0s_1,1s_1\right)$.

2. $schedule \leftarrow (\ )$

$\pi \leftarrow \{p_1,...,p_n\}$

**while** $\Pi \neq \emptyset$ **do**

**if** there is a $p_i \in \Pi$ such that there does not exist $p_j$ and dependency $\left(p_j, p_i\right)$

**then do** {append $p_i$ to *schedule*

remove all dependencies $\left(p_i, p_k\right)$ for any $p_k \in \Pi$

remove $p_i$ from $\Pi$ }

**else return** "There is a deadlock"

**return** *schedule*