CS2223
HW#1

DUE: Monday, November 3

1. (4 points) A deck of 10 cards, each bearing a distinct number from 1 to 10, is shuffled. Three cards are removed, one at a time from the deck. What is the probability that the three cards are removed in sorted (increasing) order?

2. (8 points) Do **Problem 2-4**, parts ***a***, ***b*** and ***c***, on pages 39→40 of our text.

3. (6 points) One way to evaluate a polynomial $P(x) = \sum_{0 \le i \le n} a_i x^i = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$ for a particular value of x is

$P \leftarrow a_0$

for $i \leftarrow 1$ **to** n **do**

  $xpower \leftarrow x$

  for $j \leftarrow 2$ **to** i **do**

    $xpower \leftarrow x * xpower$ (*)

  $P \leftarrow P + a_i * xpower$ (**)

We want to count the multiplications performed in the statements marked with (*) and (**).

a As a function of n, exactly how many multiplications are done in the worst-case by the preceeding code fragment?

b As a function of n, exactly how many multiplications are done in the average-case by the preceeding code fragment?

4. (8 points) For several values of n, write a program to generate an array $A[1..n]$ of n random numbers, and then compute and return the maximum value of n. Test your program and show your results.

a How much time does your program take to generate each number?

b How much time does your program take to process each number in testing for MAX?

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HW#1 SOLUTIONS

1. The experiment is equivalent to asking the probability that a random permutation (drawn from a uniform distribution) of {1,2,3} will be increasing, where 1, 2 and 3 correspond to the smallest, the median and the largest numbers drawn. Each of the $3! = 6$ permutations being equiprobable, the answer is 1/6.

2. ***a*** (1,5), (2,5), (3,4), (3,5), (4,5)

b <*n*, *n*-1,…,3,2,1> has $\sum_{n \geq j \geq 1} (j-1) = \sum_{n \geq j \geq 1} j - \sum_{n \geq j \geq 1} 1 = \frac{n(n+1)}{2} - n = \frac{n(n-1)}{2}$ inversions.

c The **do**-loop of lines 6-7 is executed exactly once for each inversion. The execution time of INSERTION-SORT has a linear factor plus a factor proportional to the number of inversions in the input array.

3. ***a*** and ***b*** both have the exact same answer, which is

$$\sum_{1 \leq i \leq n} \left(\left(\sum_{2 \leq j \leq i} 1 \right) + 1 \right) = \sum_{1 \leq i \leq n} ((i-1)+1) = \sum_{1 \leq i \leq n} i = \frac{n(n+1)}{2}$$