CS2022/MA2201

Name______________

Date: September 17, 1999
All documentation permitted

1. (8 points) Determine whether $p \rightarrow (q \rightarrow r)$ and $p \rightarrow (q \wedge r)$ are equivalent. Justify your response.

2. (12 points) Suppose that *Q*(x) is the statement $x + 1 = 2x$, where *x* is a real number. Are the following statements true or false?(a) *Q*(2)(b) $(\forall x)Q(x)$(c) $(\exists x)Q(x)$

3. (12 points) Tell whether each of the following statements are true or false for all sets *A*, *B* and *C*. If a statement is false, provide examples of sets for which the statement is false.
(*a*) If $A \cap C = B \cap C$, then *A*=*B*.

(*b*) If $A \cap B = A \cup B$, then *A*=*B*.

(*c*) $A - (B - C) = (A - B) - C$

4. (8 points) If $A=\{a,b,c\}$ and $B=\{1,\{1\}\}$, what is $|P(A\times B)|$, where $P(A\times B)$ is the power set of $A\times B$.?

5.(15 points) Show that $\{n\,|\,(\exists m\in Z^{+})\,n=10^{m}\}=\{10,100,1000,10000,...\}$ is countably infinite.

6. (9 points) For each of the following functions, either describe its inverse or else tell why the inverse doesn't exist. (*a*) $f:\mathrm{R}\rightarrow\mathrm{R}$ where $f(x)=3x-5$.

(*b*) $f:\{a,b,c\}\rightarrow\{1,2,3,4\}$ where *f*(*a*)=2, *f*(*b*)=4 and *f*(*c*)=1.

(*c*) $f:\mathrm{R}\rightarrow\mathrm{R}$ where $f(x)=\lfloor 5x\rfloor$.

7. (20 points) Prove or give a counterexample to the following:
<u>CONJECTURE</u>: For any two rational numbers *x* and *y* such that $x<y$, there is a rational number *z* such that $x<z<y$.

8. (16 points) Show that $f(n) = 20x^3 + 500x + 12$ is $O\left(x^3\right)$.

CS2022/MA2201

Solutions to Midterm Exam

1. The statements have different values when p and r are true and q is false.

2.(*a*) false (*b*) false (*c*) true

3. (*a*) false, let $A=\{1\}$, $B=\{2\}$ and $C=\varnothing$ (*b*) true (*c*) false, let $A=B=C=\{1\}$

4. $|A\times B|=6$ and $|P(A\times B)|=2^6=64$

5. Since it is not finite, $\{n \mid (\exists m\in Z^+)\, n=10^m\}$ is infinite. To show that it is countable, we consider the bijection $f: Z\rightarrow\{n \mid (\exists m\in Z^+)\, n=10^m\}$ where $f(n)=10^n$ and $f^{-1}(x)=\log_{10}x$,

which is clearly a bijection as shown by

1	2	3	4	5	6	7	8...
b	**b**	**b**	**b**	**b**	**b**	**b**	**b**
10	10^2	10^3	10^4	10^5	10^6	10^7	10^8....

6. (a) $f^{-1}(x)=\dfrac{5+x}{3}$

(*b*) *f* has no inverse because $f^{-1}(3)$ wouldn't be defined.

(*c*) *f* has no inverse because it isn't one-to-one. That is, $f(3.01)=f(3.02)=15$.

7. The Conjecture is true. For any rational numbers x and y, we can express them as ratios of integers $x=\dfrac{a}{b}$ and $y=\dfrac{c}{d}$. But $z=\dfrac{x+y}{2}=\dfrac{\frac{a}{b}+\frac{c}{d}}{2}=\dfrac{ad+bc}{2bd}$ is a rational ($ad+bc$ and $2bd$ are integers and $2bd\neq 0$), and $x<z<y$.

8. $f(n)=20x^3+500x+12\leq 20x^3+500x^3+12x^3=532x^3$ for all $x\geq 1$. Choosing $C=532$ and $k=1$, it follows that $f(n)=20x^3+500x+12$ is $O(x^3)$.