CS2022/MA2201

Name_____________

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All documentation permitted

1. (10 points) Suppose that a "word" is any string of 7 letters of our alphabet, such as *ZZRQZKA*, *TOONCES*, or *CCCCCCC*. (Our alphabet has 26 letters.)(a) How many words are there?

(b) How many words have no repeated letters?

(c) How many words end with the letter *T*?

(d) How many words begin with *R* and end with *T*?

(e) How many words begin with *A* or *B*?

2. (12 points) Our class has 120 students and 2 TAs.
(*a*) In how many ways can we form a committee of 7 people?

(*b*) In how many ways can we form a committee of 7 people if we insist that the committee contain at least one TA?

3. (10 points) Isaac picks a playing card at random from a deck of 52 cards. Consider the events E: the card is a heart

F: the card is a queen

Isaac claims that since $p(E \cup F) = p(E) + p(F)$ and $p(E) = 1/4$ and $p(F) = 1/13$, then $p(E \cup F) = p(E) + p(F) = \frac{1}{4} + \frac{1}{13} = \frac{17}{52}$. Is he right? If he is wrong, tell why?

4. (12 points) Suppose we define a relation $R = \{(a,b),(b,a),(a,a),(b,b),(c,c)\}$ over some **unknown** set A (where we only know that $\{a,b,c\} \subseteq A$). For each of the following three questions, a possible response is that we can't give a response unless we know A.

(a) Is R reflexive?

(b) Is R symmetric?

(c) Is R transitive?

5.(13 points) Consider the relation $R = \{(a,b) \mid a \text{ divides } b \text{ evenly}\}$ over Z^{+}. Is R an equivalence relation? Justify your response.

6. (9 points) Draw a graph of five vertices, each of which has degree three, or else prove that no such graph exists.

7. (18 points) Let v and w be any pair of adjacent vertices of $K_{3,3}$.
(*a*) How many paths of length 1 are there from v to w?

(*b*) How many paths of length 2 are there from v to w?

(*c*) How many paths of length 3 are there from v to w?

8. (16 points) Let sample space Ω be the set of all permutations of (1, 2, 3).
(*a*) What is $|\Omega|$?

(*b*) For any permutation $\pi \quad \Omega$, define random variable X such that $X(\pi)$ is the number of digits which are in their "proper" position in π. That is, $X(1,2,3)=3$, $X(3,1,2)=0$ and $X(1,3,2)=1$. What is the expected value of X assuming that each $\pi \quad \Omega$ is equally likely to be drawn?

<u>CS2022/MA2201</u>

Solutions to <u>Final Exam</u>

1. (*a*) 26^7 (*b*) $P(26,7)=26*25*24*23*22*21*20$ (*c*) 26^6 (*d*) 26^5 (*e*) 2×26^6

2.(*a*) $C(122,7)$
(*b*) The committee can contain 1 TA or 2 TAs. Hence,
$C(120,6)*C(2,1)+C(120,5)*C(2,2)=2C(120,6)+C(120,5)$

3. Isaac is wrong. He double counts the probability of choosing the queen of hearts.
$p(E\cup F)=p(E)+p(F) \quad p(E \quad F)=\frac{1}{4}+\frac{1}{13} \quad \frac{1}{52}=\frac{16}{52}$.

4. (*a*) We can’t give a response without knowng *A*. If $A=\{a,b,c\}$ then *A* is reflexive else *A* is not reflexive. (*b*) yes (*c*) yes

5. Although it is reflexive and transitive, *R* is not an equivalence relation because it is not symmetric. For example, $3R6$ but not $6R3$.

6. Such a graph can not exist since the sum of the degrees of the vertices of a graph must be even.

7. (*a*) 1 (*b*) 0 (*c*) 9

8. (*a*) 6 (*b*) $E[X]=1$