CS2022/MA2201

FINAL EXAM

Name______________

Date: October 16, 2003 **All documentation permitted**

1. (16 points) Let A be the set of all decimal integers. That is, $A = \bigcup_{n \geq 0} \{0,1,...,9\}^n$. For each of the following relations, tell whether or not it is reflexive, antisymmetric, symmetric or transitive.

(a) $R_1 = \{(a,b) \mid a \text{ and } b \text{ have no digits in common}\}$

reflexive

antisymmetric

symmetric

transitive

(b) $R_2 = \{(a,b) \mid a \text{ and } b \text{ have the same length}\}$

reflexive

antisymmetric

symmetric

transitive

(c) $R_3 = \{(a,b) \mid a \text{ and } b \text{ have different lengths}\}$

reflexive

antisymmetric

symmetric

transitive

(d) $R_4 = \{(a,b) \mid a \text{ has more digits than } b\}$

reflexive

antisymmetric

symmetric

transitive

2. (8 points) (a) What is the transitive closure of the $<$ relation over $\mathbb{Z}$?

(b) Prove or give a counterexample to the following:

<u>CONJECTURE</u>: Any relation R over $\mathbb{Z}$ is always a subset of its symmetric closure.

3. (10 points) A fair red die and a fair blue die are rolled. What is the expected value of the sum of the number on the red die plus 3 times the number on the blue die?

4. (15 points) (a) Draw a graph which admits an Euler circuit which does not admit a Hamilton circuit.

(b) Draw a graph which admits a Hamilton circuit which does not admit an Euler circuit.

5. (20 points) A computer picks at random a sequence of 5 digits. Let E be the event that the 5 digits are all distinct, and F be the event that the 5 digits are in weakly increasing order. That is, if the 5 digits are d_1, d_2, d_3, d_4, d_5, then F denotes the event that $d_1 \le d_2 \le d_3 \le d_4 \le d_5$.

a) What is $p(E)$?

b) What is $p(F \mid E)$?

6. (16 points) If $R = \{(a,b), (a,a), (b,c)\}$ is a relation over $A = \{a, b, c, d\}$, then what is the smallest equivalence relation containing R?

7. (15 points) Prove or give a counterexample to the following.

CONJECTURE: For any simple graph $G = (V, E)$, if $(\forall v \in V)\deg(v) = 3$, then $|V|$ is even.

CS2022/MA2201

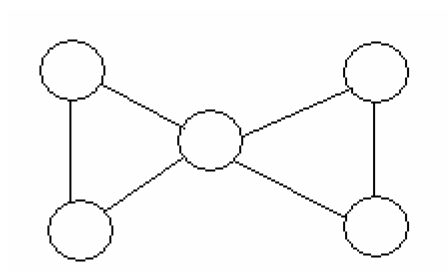
Solutions to Final Exam

1. (a) symmetric only
(b) relexive, symmetric and transitive only
(c) symmetric only
(d) antisymmetric and transitive only

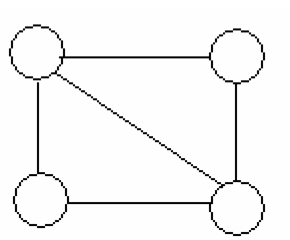
2. (a) $<$
(b) Since the symmetric closure of *R* is the smallest symmetric relation **containing** *R*, it follows that *R* must be a subset of its symmetric closure.

3. Letting rv X_r denote the number on the red die and X_b the number on the blue die, the problem asks for the value of $E[X_r + 3X_b] = E[X_r] + 3E[X_b] = 3.5 + 3*3.5 = 14$.

4. (a)



(b)



5. (a) There are $P(10,5) = 10*9*8*7*6 = 30240$ ways to choose 5 distinct digits, and 10^5 ways to choose 5 digits. So $p(E) = \frac{P(10,5)}{10^5} = .3024$.
(b) Having chosen 5 distinct digits, there are $P(5,5) = 5! = 120$ ways to permute them. So

$$p(F \mid E) = \frac{1}{P(5,5)} = .0083$$

6. $\{a,b,c\} \times \{a,b,c\} \cup \{(d,d)\}$

7. The CONJECTURE is true. As we saw in class (and as shown in THEOREM 1 on pg. 546 of our text), $\sum_{v \in V} \deg(v)$ is even. Since every vertex has degree 3, this can only be true if $|V|$ is even.