CS2022/MA2201

Final Exam

Name______________

Date: October 18, 2001
All documentation permitted

1. (15 points) Let relation *R* correspond to the array

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

a Is *R* reflexive?

b Is *R* symmetric?

c Is *R* antisymmetric?

d Is *R* transitive?

e Is *R* an equivalence relation?

2. (15 points) Suppose that a chip fabrication facility has a probability of .01 of producing a defective chip, and manufacturing defects are independent. What is the probability that in a batch of 100 chips at most 3 are defective?

3. (10 points) How many bit strings of length ten have an equal number of 0s and 1s? For example, 0011110010 should be counted.

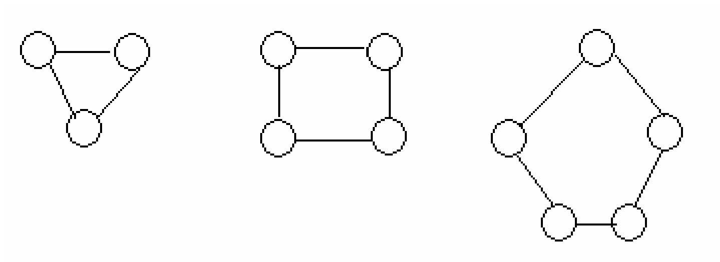
4. (25 points) ***a*** A Computer Science department has 10 male faculty members and 10 female faculty members. A committee of 4 people is chosen randomly (each set of 4 people is equally likely to be chosen). What is the probability that the committee consists of only females?

b What is the probability that the committee consists of only females or only males?

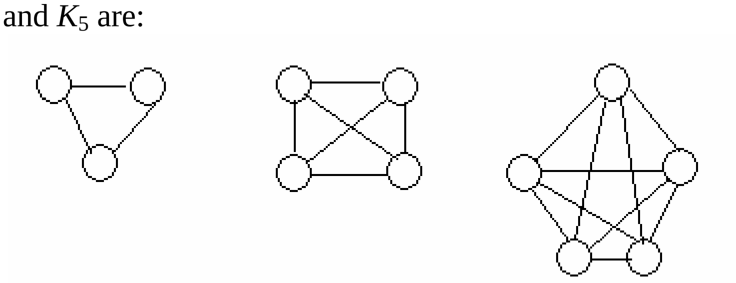
5. (15 points) Either prove or give a counterexample to the following:

<u>CONJECTURE</u>: For every simple graph $G = (V, E)$, if $|V| \geq 3$ and $(\forall v \in V)\deg(v) \geq \frac{|V|}{2}$, then G has an Euler circuit. That is, if every vertex of G is adjacent to at least half the vertices of G, then G must be Eulerian.

6. (20 points) ***a*** For which values of $n \geq 3$ is C_n bipartite, where C_n is a cycle of n vertices? For example, C_3, C_4 and C_5 are:



b For which values of $n \geq 3$ is K_n bipartite, where K_n is a complete graph of n veritices? For example, K_3, K_4 and K_5 are:



CS2022/MA2201

Solutions to Final Exam

1. ***a*** yes ***b*** no ***c*** yes ***d*** yes ***e*** no

2. Letting k count the number of of successes (non defective chips),

$$\sum_{97\le k\le 100}\binom{100}{k}.99^{k}.01^{100-k} = .9816259635$$

3. There are $\binom{10}{5}=252$ ways to choose positions for the five 0s among the ten positions.

4. ***a*** There are $\binom{20}{4}$ possible committees, and there are $\binom{10}{4}$ committees consisting only of women. The probability of choosing a committee of only women is $\dfrac{\binom{10}{4}}{\binom{20}{4}} = \dfrac{14}{323}$

b The probability for a committee of 4 men is the same. Since the events "committee of all women" and "committee of all men" are disjoint, the probability of their union is the sum of the probabilities, or $\dfrac{28}{323}$.

5.The CONJECTURE is false. A counterexample is the graph K_4 in which every vertex has odd degree.

6. ***a*** $\{4,6,8,...\}=\{2n+2 | n\in\mathbb{Z}^{+}\}$ ***b*** $\varnothing$