CS2022/MA2201

Name________

Midterm Exam

Date: September 22, 1997

All documentation permitted

1. (5 points) Is $(\neg q \vee p) \Leftrightarrow (q \rightarrow p)$? Justify your response.

2. (12 points) Letting the Universe of Discourse be the set of students at WPI, let $P(x)$ denote the statement "x likes CS2022/MA2201". Express, in predicate calculus, the sentence " At least two students like CS2022/MA2201, though not everybody likes it".

3. (12 points) Suppose that $|A|=m=1$ and $|B|=n=1$. What is the most that can be said about the relationship between m and n for each of the following to be true?

a) There is an injection from A to B.

b) There is a surjection from A to B.

c) There is a bijection from A to B.

4. (21 points) A) Translate the following inference into propositional logic.

> If today is Thursday, then I have a test in CS or a test in Econ. If my Econ professor is sick, then I will not have a test in Econ. Today is Thursday and my Econ professor is sick. Therefore I have a test in CS.

B) Is the inference correct? Justify your response.

5. (30 points) An *irrational number* is a number which is not rational, that is, it can not be expressed as the ratio of two integers. Prove or give a counterexample to the following:
<u>CONJECTURE</u>: For all rational numbers x, x?0, and irrational numbers y, the product of x and y must be irrational.

(Hint: The conjecture states that it's not possible for there to be a rational number $x*y$. If the CONJECTURE is true, then you must prove the no such rational number exists. Think of how we prove a negative result like "such a rational number can't exist". If it's false, you only need demonstrate a rational x?0 and an irrational y such that $x*y$ is rational.)

6. (20 points) Express the sum $\sum_{k=o}^{n} x^k$ using O-notation, in the form $O(x^m)$, where m is the smallest integer for which the statement is true.

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Solutions to Midterm Exam

1. A truth table establishes the logical equivalence.

q	p	$\neg q$	$(\neg q \vee p)$	$(q \rightarrow p)$
F	F	T	T	T
F	T	T	T	T
T	F	F	F	F
T	T	F	T	T

2. $(\exists x)(\exists y)\big(P(x) \wedge P(y) \wedge (x \neq y)\big) \wedge \neg(\forall x)P(x)$

3. ***a***) *m=n*
b) *m=n*
c) *m=n*

4. t - Today is Thursday.
c - I have a test in CS.
e - I have a test in Econ.
s - My econ professor is sick.

The argument is:

$t \rightarrow (c \vee e)$

$s \rightarrow \neg e$

$t \wedge s$

$\therefore c$

From $t \wedge s$ we can conclude *t* and *s* (simplification). From *t* and $t \rightarrow (c \vee e)$ we can conclude $c \vee e$ (modus ponens). From *s* and $s \rightarrow \neg e$ we can conclude $\neg e$ (modus ponens). From $c \vee e$ and $\neg e$ we can conclude *c* (disjunctive syllogism).

5. The conjecture is true. (Proof by Contradiction) If it were false, there'd be a rational $x=a/b$, $a,b \in Z$ and an irrational *y* such that there exist $c,d \in Z$ such that $x*y$ is rational, that is, $x*y=a*y/b=c/d$. But this implies $y=b*c/(a*d)$. But since $b*c \in Z$ and $a*d \in Z$, this would imply that *y* is a rational number, which is a contradiction.

6. $\displaystyle\sum_{k=o}^{n} x^k = \frac{x^{n+1}-1}{x-1} = O(x^n)$