CS2022/MA2201
HW#8 Solutions

1)

	reflexive	symmetric	antisymmetric	transitive
a)	n	n	y	y
b)	y	y	n	y
c)	y	y	n	y
d)	y	y	n	n

2) **a**) *Course* **b**) *Course Number* **c**) *Course Number* **d**) *Departure Time*

3)

Airline	***FlightNumber***	***Destination***
Nadir	122	Detroit
Acme	221	Denver
Acme	122	Anchorage
Acme	323	Honolulu
Nadir	199	Detroit
Acme	222	Denver
Nadir	322	Detroit

4) $Z \times Z$

5) yes

6) {010, 01010101} {1011} {11111}

7) **a**) yes **b**) no $0 \notin$ a class **c**) yes **d**) no -110 belongs to two classes
e) no 8 belongs to 2 classes

8) The conjecture is true.
PROOF: Let $|V|=n$. The possible degrees of the n vertices are $0,\dots,n$-1. It is impossible to have vertices of degree n-1 **and** 0, because a vertex of degree n-1 would be adjacent to every other vertex, and it couldn't be adjacent to a vertex of degree 0. Since this implies that the n vertices can have at most n-1 degrees, by the pigeonhole principle at least two vertices must have the same degree.

9) The conjecture is true.
PROOF: Let P be a longest path in G, and assume the length of P is less than δ, and let v be an endpoint of P. Because P is a longest path, each of v's neighbors must be on P (or else P could be extended by adding a neighbor of v to the end of P. But since v has at least δ neighbors and each of these neighbors is on P, the length of P must be at least δ. By contradiction, G must have a path of length at least δ.