CS2022/MA2201
HW#8 Solutions

1) (*a*) $C\left(n,0\right)p^0\left(1-p\right)^{n-0}=\left(1-p\right)^n$

(*b*) $\sum_{k=1}^{n}C\left(n,k\right)p^k\left(1-p\right)^{n-k}$. Since this event is the complement of the event of having exactly 0 successes, its probability is $1-\left(1-p\right)^n$.

(*c*) This is the probability that there are 0 successes or 1 success, which is
$C\left(n,0\right)p^0\left(1-p\right)^{n-0}+C\left(n,1\right)p^1\left(1-p\right)^{n-1}=\left(1-p\right)^n+np\left(1-p\right)^{n-1}$

(*d*) $\sum_{k=2}^{n}C\left(n,k\right)p^k\left(1-p\right)^{n-k}$. Since this is the probability that there aren't exactly 0 or 1 successes, it is equivalent to $1-\sum_{k=0}^{1}C\left(n,k\right)p^k\left(1-p\right)^{n-k}=1-\left(1-p\right)^n-np\left(1-p\right)^{n-1}$.

2) There are *C*(50,6) possible tickets, and each ticket is equally likely. Hence, the probability of choosing any ticket is $\frac{1}{C\left(50,6\right)}$. Defining rv *X* to denote the winnings on any ticket, the problem asks for E[*X*], where $X\left(\omega\right)=10{,}000{,}000$ for exactly one ω and $X\left(\omega\right)=0$ for all other tickets. By the definition,

$$E\left[X\right]=\sum_{\omega}X\left(\omega\right)p\left(\omega\right)=\frac{10{,}000{,}000}{C\left(50,6\right)}=\frac{10{,}000{,}000}{15{,}890{,}700}\quad \$0.63$$ (not worth the $1 price).

3) (*a*) 1,3,4 (*b*) 1 (*c*) 1,2 (*d*) 1,2,4 (*e*) 1,3,4 (*f*) 2 (*g*) 2 (*h*) 1,2,4 (*i*) 1,2,4 (*j*) 1,3,4 (*k*) 1,3,4 (*l*) 3,4 (*m*) 1,2 (*n*) 1,2,4.

4) (a) {(a,a),(a,c),(b,c),(c,c),(d,b),(d,d)}
(b) {(a,b),(a,c),(a,d),(b,c),(c,c),(d,a),(d,c)}
(c) {(b,a),(d,c)}
(d) {(b,c)}
(e) {(a,c),(b,a),(d,b),(d,d)}
(f) {(a,a),(a,d),(d,c)}.

5)

(a) $\begin{bmatrix}1 & 0 & 0\\ 1 & 1 & 1\\ 1 & 0 & 1\end{bmatrix}$ (b) $\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & 1 & 1 & 1\\ 1 & 1 & 1 & 1\\ 1 & 1 & 1 & 1\end{bmatrix}$