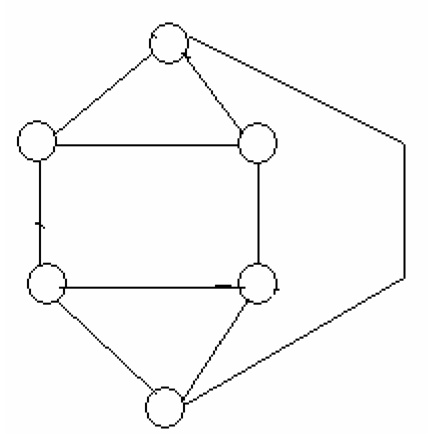
CS2022/MA2201
HW#8 SOLUTIONS

1. ***a*** This is an equivalence relation.
b This is an equivalence relation.
c This is not an equivalence relation since it is not transitive.
d This is not an equivalence relation since it is not transitive.
e This is not an equivalence relation since it is not transitive.

2. This is an equivalence relation.

3. ***a*** There is none, since the number of vertices of odd degree must be even.
b The following graph is planar, so it can't be isomorphic to $K_{3,3}$.



c There is none. If $|E| = 8$, then $\sum_{v\in V}\deg(v) = 2|E| = 16$. But in a cubic graph, each vertex has degree 3, so $\sum_{v\in V}\deg(v) = 3|V| = 16$ which implies that $|V| = \frac{16}{3}$, which is impossible.

4. The graphs are isomorphic, and one isomorphism is

$$f(u_1) = v_1,\ f(u_2) = v_2,\ f(u_3) = v_4,\ f(u_4) = v_5,\ f(u_5) = v_3$$

5. The graphs can not be isomorphic since the second graph has a vertex of degree 4 and the first graph does not (violating a necessary condition for isomorphism).

6. We note that for any graph G and for any $v \in V$, $0 \le deg(v) \le |V| - 1$. We consider two cases:

- $(\exists v \in V)\, deg(v) = |V| - 1$

  Since every vertex is adjacent to v, then the degrees of all vertices $w \in V$ satisfy $1 \le deg(w) \le |V| - 1$. Since there are n vertices and only n-1 possible degrees, by the pigeonhole principle there must be at least two vertices of the same degree.

- $\neg(\exists v \in V)\, deg(v) = |V| - 1$

  Since no vertex is adjacent to every other vertex, then the degrees of all vertices $w \in V$ satisfy $0 \le deg(w) \le |V| - 2$. Since there are n vertices and only n-1 possible degrees, by the pigeonhole principle there must be at least two vertices of the same degree.