CS2022/MA2201
HW#8

DUE: Monday, April 30

1. (8 points) Adapted from Rosen. Consider the following relations:

$$R_1 = \{(x, y) \in \mathbb{Z}^2 \mid x > y\}$$
$$R_2 = \{(x, y) \in \mathbb{Z}^2 \mid x \geq y\}$$
$$R_3 = \{(x, y) \in \mathbb{Z}^2 \mid x < y\}$$
$$R_4 = \{(x, y) \in \mathbb{Z}^2 \mid x \leq y\}$$
$$R_5 = \{(x, y) \in \mathbb{Z}^2 \mid x = y\}$$
$$R_6 = \{(x, y) \in \mathbb{Z}^2 \mid x \neq y\}$$

a What is $R_1 \cup R_3$?
b What is $R_1 \cup R_5$?
c What is $R_2 \cap R_4$?
d What is $R_3 \cap R_5$?
e Is $R_3 \cap R_5$ transitive?
f What is $R_2 - R_1$?
g Is $R_5 - R_6$ finite, countably infinite or uncountably infinite?
h What is the reflexive closure of R_1?

2. (8 points) For each of the following claims, tell whether it is true or false and justify your answer.
a A relation R on a set A can never be symmetric and antisymmetric.
b If a relation R on a set A is reflexive and symmetric, then it must be transitive.
c Every relation is symmetric or antisymmetric or both.
d For any equivalence relation R, the cardinality of the transitive closure of R is equal to the cardinality of R.

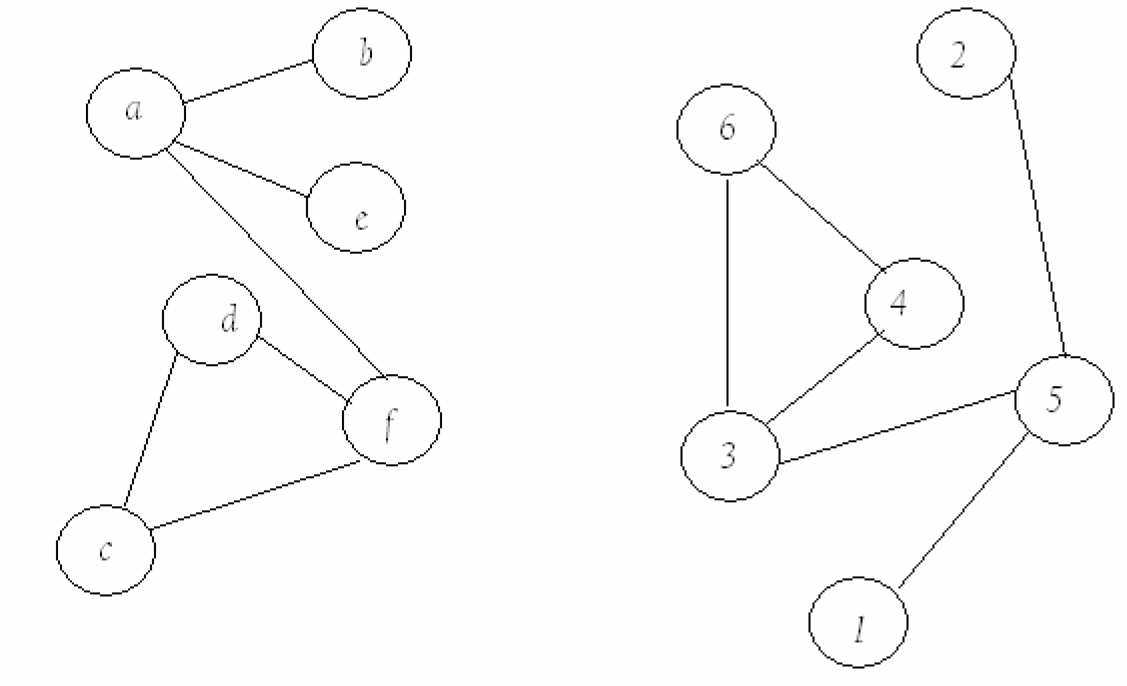
3. (8 points) Let R_1 and R_2 be equivalence relations over set A. We say that R_1 is a *refinement* of R_2 if $\forall x \in A \forall y \in A(xR_2 y \rightarrow xR_1 y)$. Now let R^5 and R^9 be defined over the set of all strings of at least 9 bits such that $xR^5 y$ if the first 5 bits of x are the same as the first 5 bits of y and $xR^9 y$ if the first 9 bits of x are the same as the first 9 bits of y.
a Are R^5 and R^9 equivalence relations? Justify your answer.
b If your answer to part ***a*** is yes, then

- Is R^5 a refinement of R^9? Justify your answer.
- Is R^9 a refinement of R^5? Justify your answer.

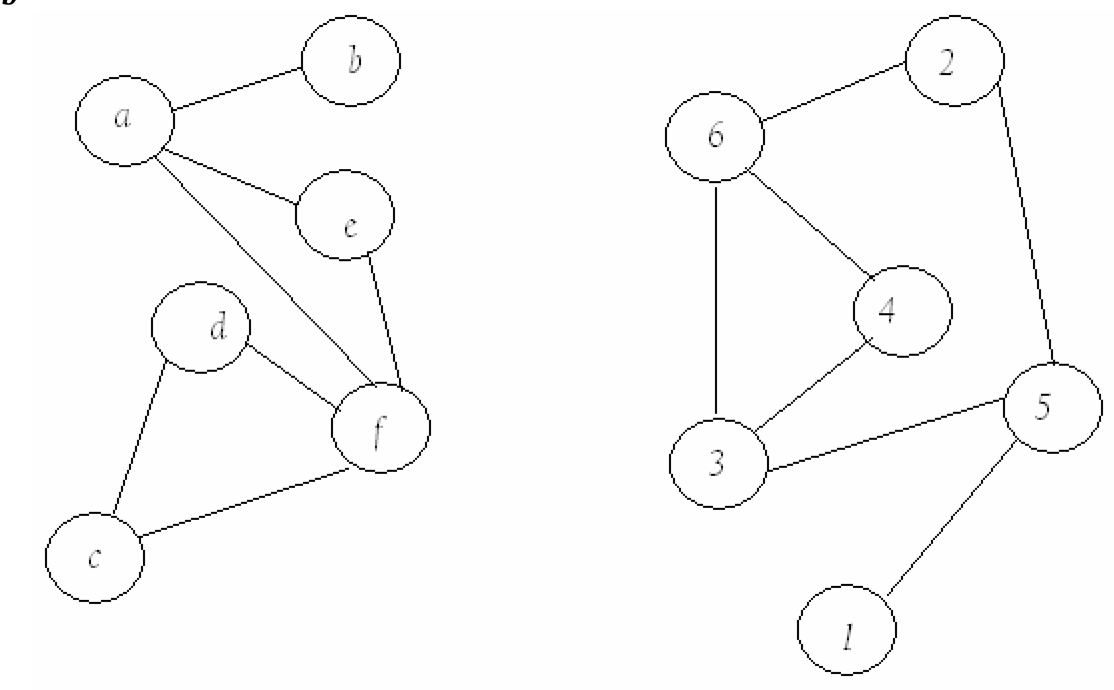
If the correct answer to part ***a*** was no, then you all get credit for part ***b***.

4. (5 points) Are the following pairs of graphs isomorphic? Justify your answers.

a



b



5. (8 points) ***a*** For which values of *m* and *n* does $K_{m,n}$ have an Euler circuit?

b For which values of *m* and *n* does $K_{m,n}$ have an Euler path?

c For which values of *m* and *n* does $K_{m,n}$ have an Hamilton circuit?

d For which values of *m* and *n* does $K_{m,n}$ have an Hamilton path?

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HW#8 SOLUTIONS

1. ***a*** $R_1 \cup R_3 = R_6$.
b $R_1 \cup R_5 = R_2$.
c $R_2 \cap R_4 = R_5$.
d $R_3 \cap R_5 = \varnothing$.
e $R_3 \cap R_5$ is transitive.
f $R_2 - R_1 = R_5$.
g $R_5 - R_6 = R_5$ is countably infinite, because it admits the bijection

$$\begin{array}{ccccccc} (0,0) & (-1,-1) & (1,1) & (-2,-2) & (2,2) & (-3,-3) & (3,3)\ldots \\ | & | & | & | & | & | & | \\ 1 & 2 & 3 & 4 & 5 & 6 & 7\ldots \end{array}$$

h R_2

2. ***a*** The claim is false since the relation $\{(1,1)\}$ over the set {1} is symmetric and antisymmetric.
b The claim is false since the relation $R=\{(1,1),(2,2),(3,3)\}$ over the set {1,2,3} is reflexive and symmetric but not transitive (because $(1,2),(2,3)\in R$ but $(1,3)\notin R$).
c The claim is false, since the relation $R=\{(1,2),(2,3),(3,2)\}$ is neither symmetric (because $(1,2)\in R$ but $(2,1)\notin R$) nor antisymmetric (because $(2,3),(3,2)\in R$ but $2\neq 3$).
d The claim is true, since an equivalence relation must be transitive, and hence it is equal to its transitive closure.

3. ***a*** R^5 and R^9 are equivalence relations. Every bit string agrees with itself on every prefix, and if bit string *x* agrees with bit string *y* on its first 5 (or 9) bits, then clearly *y* agrees with *x* on its first 5 (or 9) bits. Finally, if bit string *x* agrees with bit string *y* on its first 5 (or 9) bits and *y* agrees with *z* on its first 5 (or 9) bits, then *x* must agree with *z* on its first 5 (or 9) bits.
b • R^5 a not refinement of R^9 since $(0000011111, 0000011011)\in R^5$ but $(0000011111, 0000011011)\notin R^9$.
• R^9 a not refinement of R^5 since any two bit strings that agree on their first 9 bits must also agree on their first 5 bits.

4. ***a*** They are isomorphic under the bijection

$$\begin{array}{cccccc} a & b & c & d & e & f \\ | & | & | & | & | & | \\ 5 & 2 & 1 & 4 & 6 & 3 \end{array}$$

b They can not be isomorphic since the first graph contains a vertex of degree 4 and the second graph does not.

5. ***a*** m and n both even

b (m and n both even) or (m=n=1) or (n=2)

c m=n

d (m=n) or (m=n+1)