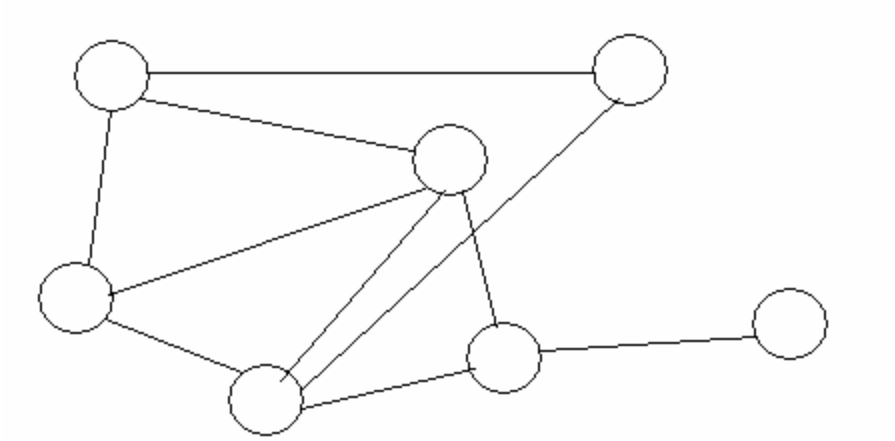
CS2022/MA2201
HW#8

DUE: Tuesday, October 14 (Covers graphs.)

1. (8 points) For which values of n are these graphs bipartite?
   (a) K_n (b) C_n (c) W_n (d) Q_n
(Note that this is **Exercise 8.2-24** of our text.)

2. (6 points) For any graph G with v vertices and e edges, let $\Delta(G)$ be the maximum degree of all vertices of G and let $d(G)$ be the minimum degree of all vertices of G. For the graph G



v=7, e=10, $\Delta(G)=4$ and $d(G)=1$. We can observe that for this graph $2e/v = 20/7$ and $d(G) \le 2e/v \le \Delta(G)$. Is this always true? Prove or give a counterexample to

CONJECTURE For any simple graph G, $d(G) \le 2e/v \le \Delta(G)$.

3. (4 points) Draw pictures of all the nonisomorphic simple graphs on 4 vertices.

4. (4 points) For which values of n do the following graphs admit an Euler circuit?
   (a) K_n (b) C_n (c) W_n (d) Q_n

5. (8 points) For which values of n do the following graphs admit a Hamilton circuit?
   (a) K_n (b) C_n (c) W_n (d) Q_n

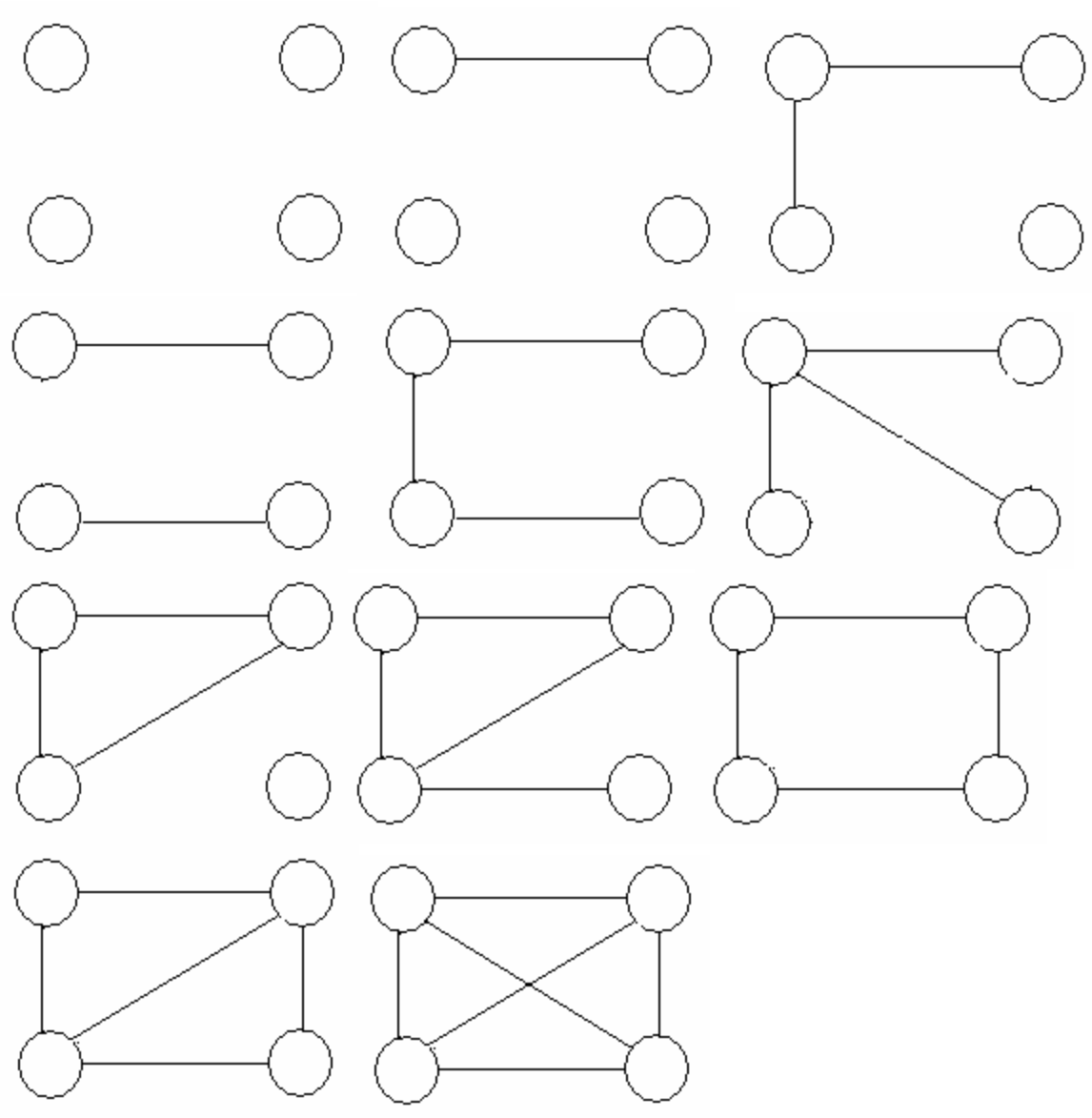
CS2022/MA2201
HW#8 SOLUTIONS

1. (a) n=1 or n=2 (b) even values of n (c) no values of n (d) $n \geq 1$

2. The CONJECTURE is true. We note that each of the e edges contributes to the degree of two vertices (the endpoints of the edge). Hence $\sum_{v \in V} \deg(v) = 2e$ and the average degree is $2e/v$.

Since the average degree must be no less than the minimum degree and no greater than the maximum degree, $d(G) \leq 2e/v \leq \Delta(G)$.

3.



4. (a) odd $n \geq 3$ (b) $n \geq 3$ (c) no values of n (d) even $n \geq 2$

5. (a) $n \geq 3$ (b) $n \geq 3$ (c) $n \geq 3$ (d) $n \geq 2$