CS2022/MA2201
HW#7 SOLUTIONS

1) *a*) *independent*
b) *dependent*
c) *dependent*

2) Since $p(E \cup F) = p(E) + p(F) - p(E \cap F) = 0.8 + 0.6 - p(E \cap F) = 1.4 - p(E \cap F)$, and $p(E \cup F) \leq 1$, it follows that $p(E \cap F) \geq 0.4$

3) Fixing the first person's birthday, the probability that it's the second person's birthday is 1/366.

4) 1/16

5) Each bit string occurs with probability 1/8. E={100, 010, 001, 111} and $p(E)$ =1/2. $p(F)$=1/2. $E \cap F = \{100{,}111\}$ and $p(E \cap F) = 1/4 = p(E)\,p(F)$, so E and F are independent.

6) *a*) $C(n,0)p^0(1\text{-}p)^n = (1\text{-}p)^n$

b) $\sum_{k=1}^{n} C(n,k)\,p^k(1-p)^{n-k} = 1 - C(n,0)\,p^0(1-p)^{n-0} = 1-(1-p)^n$

c) $\sum_{k=0}^{1} C(n,k)\,p^k(1-p)^{n-k} = C(n,0)\,p^0(1-p)^n + C(n,1)\,p(1-p)^{n-1} = (1-p)^n + np(1-p)^{n-1}$

d) $\sum_{k=2}^{n} C(n,k)\,p^k(1-p)^{n-k} = 1 - \sum_{k=0}^{1} C(n,k)\,p^k(1-p)^{n-k} = 1-(1-p)^n - np(1-p)^{n-1}$

7) From **EXAMPLE 14** of pg. 278 of our text, the expected number of successes is np=6.