CS2022/MA2201
HW#7 SOLUTIONS

1. The CONJECTURE is true. If R and S are reflexive, then $\{(a,a) \mid a \in A\} \subseteq R$ and $\{(a,a) \mid a \in A\} \subseteq S$. Therefore, $\{(a,a) \mid a \in A\} \subseteq R \cup S$, so $R \cup S$ is reflexive.

2. The CONJECTURE is true. If R is reflexive, then $\{(a,a) \mid a \in A\} \subseteq R$. Therefore, $\{(a,a) \mid a \in A\} \subseteq R \cup S$, so $R \cup S$ is reflexive.

3. The CONJECTURE is false. Let $A = \{a,b,c,d\}$, $R = \{(a,b),(b,c),(a,c)\}$ and $S = \{(b,c),(c,d),(b,d)\}$. $R \cup S$ is not transitive since $(a,c) \in R \cup S$ and $(c,d) \in R \cup S$ but $(a,d) \notin R \cup S$.

4. The CONJECTURE is false. Let $A = \{a,b,c\}$ and $R = \{(a,b),(b,a),(b,c),(c,b),(a,a),(b,b),(c,c)\}$. R is not transitive since $(a,b) \in R$ and $(b,c) \in R$ but $(a,c) \notin R$.

5. The reflexive closure of R is $\mathbb{Z} \times \mathbb{Z}$.

6.

a $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

b $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$