CS2022/MA2201
HW#7

DUE: Monday, April 23

1. (3 points) Assume that there are 65 students in our class, and I draw 3 people randomly. Also assume that none of them were born in a leap year. What is the probability that none of them have the same birthday?

2. (10 points) Let $(a_1,...,a_{10})$ be any random permutation of $(1,...,10)$ (drawn from a uniform distribution over all permuations of $(1,...,10)$). Let E_1 denote the event that $a_6 = 6$, and E_2 denote the event that $(a_1,...,a_{10}) = (1,...,10)$.

a What is $p(E_1)$?

b What is $p(E_2)$?

c What is $p(E_1 \cap E_2)$?

d What is $p(E_1 \cup E_2)$?

e Are events E_1 and E_2 independent?

3. (4 points) Suppose that a packet sent over a network has a probability of 0.8 of arriving at its destination, and that 10 packets are sent. Finally, suppose that the probabilities of arriving are independent. What is the probability that at least 8 of the 10 packets arrive at the destination? Give a numerical answer.

4. (6 points) Suppose a pair of honest dice is thrown and you know that at least one of the two dice has a face value of 6. Would this information increase or decrease the probability that the sum of the two faces is 7? Justify your answer.

5. (6 points) Suppose that for a lottery ticket you must pick 6 distinct numbers from the set $\{1, 2,..., 50\}$. The state picks 6 winning numbers by drawing 6 balls randomly (from a uniform distribution), without replacement, from a set of 50 balls, with labels 1, 2,...,50. If **all** 6 of the numbers on your ticket are winning numbers, then you win $1,000,000. If at least one of the numbers on your ticket is not a winning number, then you win $0. What is the value of your lottery ticket?

CS2022/MA2201
HW#7 SOLUTIONS

1. Fixing any order among the people, the probability that the second person's birthday is different than the first person's birthday is $\frac{364}{365}$. Assuming that the first 2 birthdays are different, the probability that the third person's birthday is different than the first 2 people's birthdays is $\frac{363}{365}$. So the answer is $\frac{364}{365} * \frac{363}{365} \approx 0.9917958341$

2. ***a*** $p(E_1) = \frac{1}{10}$

b $p(E_2) = \frac{1}{P(10,10)} = \frac{1}{10!} \approx 0.0000002755731922$

c Since $E_2 \subseteq E_1$, $p(E_1 \cap E_2) = p(E_2) = \frac{1}{10!}$.

d $p(E_1 \cup E_2) = p(E_1) + P(E_2) - p(E_1 \bigcap E_2) = \frac{1}{10}$.

e E_1 and E_2 are not independent because $p(E_1 \cap E_2) = \frac{1}{10!} \neq p(E_1) * p(E_2) = \frac{1}{10} * \frac{1}{10!}$. Certainly knowing that E_2 occurred yields information about whether E_1 occurred.

3. The answer is the probability that exactly 8 packets arrive, plus the probability that exactly 9 packets arrive, plus the probability that exactly 10 packets arrive. This probability, from the binomial distribution, is

$$\sum_{8 \le k \le 10} C(10,k).8^k.2^{10-k} = C(10,8)(.8)^8(.2)^2 + C(10,9)(.8)^9(.2)^1 + C(10,10)(.8)^{10}(.2)^0 \approx 0.678$$

4. Let E be the event that at least one of the two dice is a 6, and let F be the event that the sum of the two faces of the dice is 7. So

$E = \{(1,6),(2,6),(3,6),(4,6),(5,6),(6,6),(6,1),(6,2),(6,3),(6,4),(6,5)\}$,

$F = \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$ and $E \cap F = \{(1,6),(6,1)\}$. So $|S| = 36$,

$p(E) = \frac{11}{36}$, $p(F) = \frac{1}{6}$ and $p(F \mid E) = \frac{p(E \cap F)}{p(E)} = \frac{2/36}{11/36} = \frac{2}{11}$. Hence, since $p(F \mid E) > p(F)$, our chances of having thrown a 7 have increased by $.0\overline{15}$.

5. There are $C(50,6)$ ways for the state to draw 6 winning numbers, and this is our sample space. Exactly 1 of these elementary events corresponds to your ticket being a winning ticket. Hence, the probability of your ticket being a winning ticket is $\frac{1}{C(50,6)}$.

The expected value of the ticket is $\$1000000 * \frac{1}{C(50,6)} + \$0 * \frac{C(50,6)-1}{C(50,6)} \approx \$0.63$. If you pay $1 for the ticket you're wasting your money.