CS2022/MA2201
HW#7

DUE: Thursday, October 9 (Covers relations, Chapter 7.)

1. (8 points) Do **Exercise 7.1-4** on pg. 480 of our text.

2. (16 points) Do **Exercise 7.1-6** on pg. 480 of our text.

3. (8 points) Assume *R* and *S* are each binary relations from set *A* to *A*. Tell whether each of the following is true or false. If a statement is false, then give a counterexample.
(a) If *R* and *S* are reflexive, then $R \cap S$ is reflexive.
(b) If *R* and *S* are transitive, then $R \cap S$ is transitive.
(c) If *R* and *S* are transitive, then $R \cup S$ is transitive.
(d) If *R* and *S* are symmetric, then $R \circ S$ is symmetric.

4. (3 points) Do **Exercise 7.4-34** on pg. 507 of our text.

5. (2 points) What is the reflexive closure of the binary relation $R = \{(b,c),(b,d),(c,b)\}$ on $A=\{a,b,c,d\}$?

6. (3 points) What is the transitive closure of the binary relation $R = \{(b,c),(b,d),(c,b)\}$ on $A=\{a,b,c,d\}$?

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HW#7 SOLUTIONS

1. (a) R is not reflexive (I am not taller than myself), not symmetric (I am taller than Professor Agu, but he is not taller than I am), though it is antisymmetric and transitive.
(b) R is reflexive, symmetric and transitive, but it is not antisymmetric since different people can be born the same day.
(c) Same answer as part (b).
(d) R is reflexive and symmetric, but is not antisymmetric since my sons Ben and Isaac share a grandparent but are different people. It is not transitive since my cousin Leon and I share a grandparent, and Leon has other cousins through his other grandparents who are not my cousins.

2. (a) It is not reflexive ($42+42\neq 0$), though it is symmetric, $(\forall x)(\forall y)\, x+y=y+x$. It is not antisymmetric (since $(42,-42)\in R,(-42,42)\in R$ though $42\neq -42$) nor transitive (since $(42,-42)\in R,(-42,42)\in R$ though $(42,42)\notin R$).

(b) It is reflexive, symmetric and transitive, though not antisymmetric (since $(42,-42)\in R$).

(c) It is reflexive (since x-x=0$\in\mathbb{Q}$), symmetric and transitive, though not antisymmetric (since $(42,-42)\in R,(-42,42)\in R$ though $42\neq -42$).

(d) It is antisymmetric, though not reflexive ($(3,3)\notin R$), symmetric ($(6,3)\in R,(3,6)\notin R$) or transitive $((12,6)\in R,(6,3)\in R,(12,3)\notin R)$.

(e) It is reflexive and symmetric, though not antisymmetric $((3,5)\in R,(5,3)\in R,3\neq 5)$ or transitive $((-3,0)\in R,(0,5)\in R,(-3,5)\notin R)$.

(f) It is symmetric, though not reflexive $((3,3)\notin R)$, antisymmetric $((3,0)\in R,(0,3)\in R,3\neq 0)$ or transitive $((3,0)\in R,(0,5)\in R,(3,5)\notin R)$.

(g) It is antisymmetric and transitive, though it is not reflexive $((2,2)\notin R)$ or symmetric $((1,3)\in R,(3,1)\notin R)$.

(h) It is symmetric, though it is not reflexive $((2,2)\notin R)$, antisymmetric $((1,3)\in R,(3,1)\in R,1\neq 3)$ or transitive $((3,1)\in R,(1,5)\in R,(3,5)\notin R)$.

3. (a) true

(b) true

(c) false Let $A=\{a,b,c\}$. $R=\{(a,b)\}$ is transitive and $S=\{(b,c)\}$ is transitive, but $R\cup S=\{(a,b),(b,c)\}$ is not transitive.

(d) false Let $A=\{a,b,c\}$. $R=\{(b,c),(c,b)\}$ is symmetric and $S=\{(a,b),(b,a)\}$ is symmetric, but $(a,c)\in R\circ S$ although $(c,a)\notin R\circ S$.

4. At the bottom of the program, add the statement

   for *i*:=1 **to** *n* w_{ii}:=1

5. $R=\{(b,c),(b,d),(c,b),(a,a),(b,b),(c,c),(d,d)\}$

6. $R=\{(b,c),(b,d),(c,b),(c,c),(c,d),(b,b)\}$